Energy, energy-flux, and control

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□ Control of distributed system: more design than control

□ Non-stationary dynamical systems (stability, convergence)

□ Boundary conditions, input-output, partial integration

□ Iteration $\rightarrow$ evolution-with-damping

in collaboration with Arjan van der Schaft and Peter Breedveld

Once a system is “designed,” control u is limited

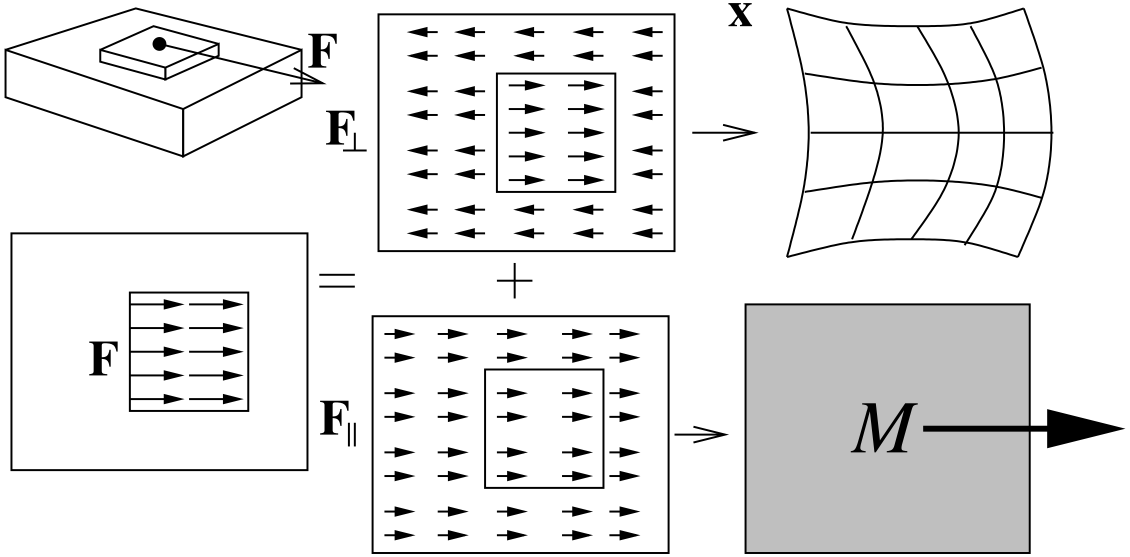
$$u(x,t) = F_{\text{design}}(x) \quad u_{\text{control}}(t)$$

E.g., in reducing vibrations, design is:

traditional: lighter and stiffer ($\omega \uparrow$)

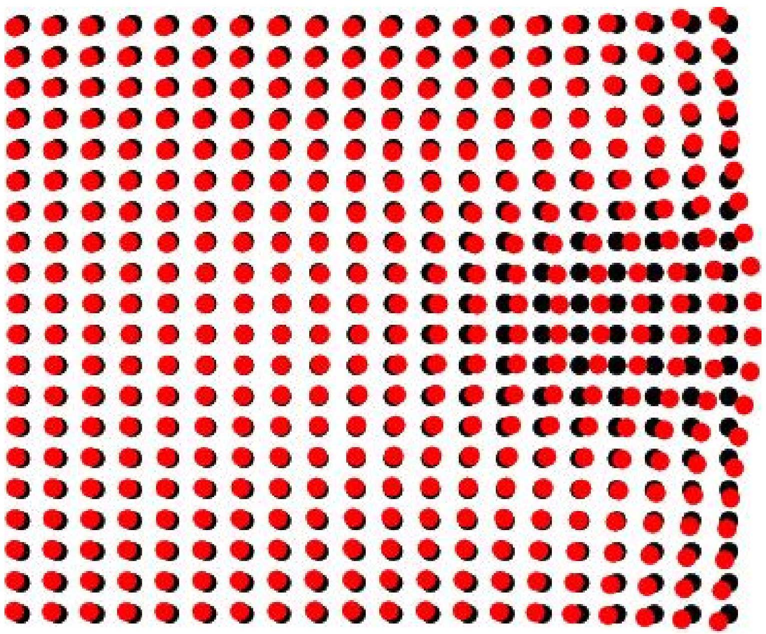
control-based: optimize input-to-output

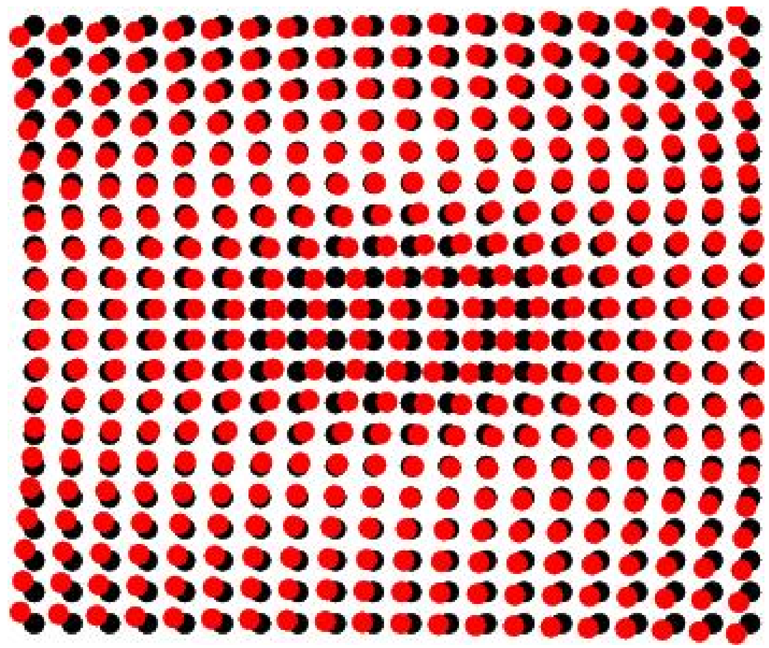
vibration is energy “lost” in the system



design control: position of the fixture. Dynamical control: time-dependence of the applied force. However, for every fixture independently

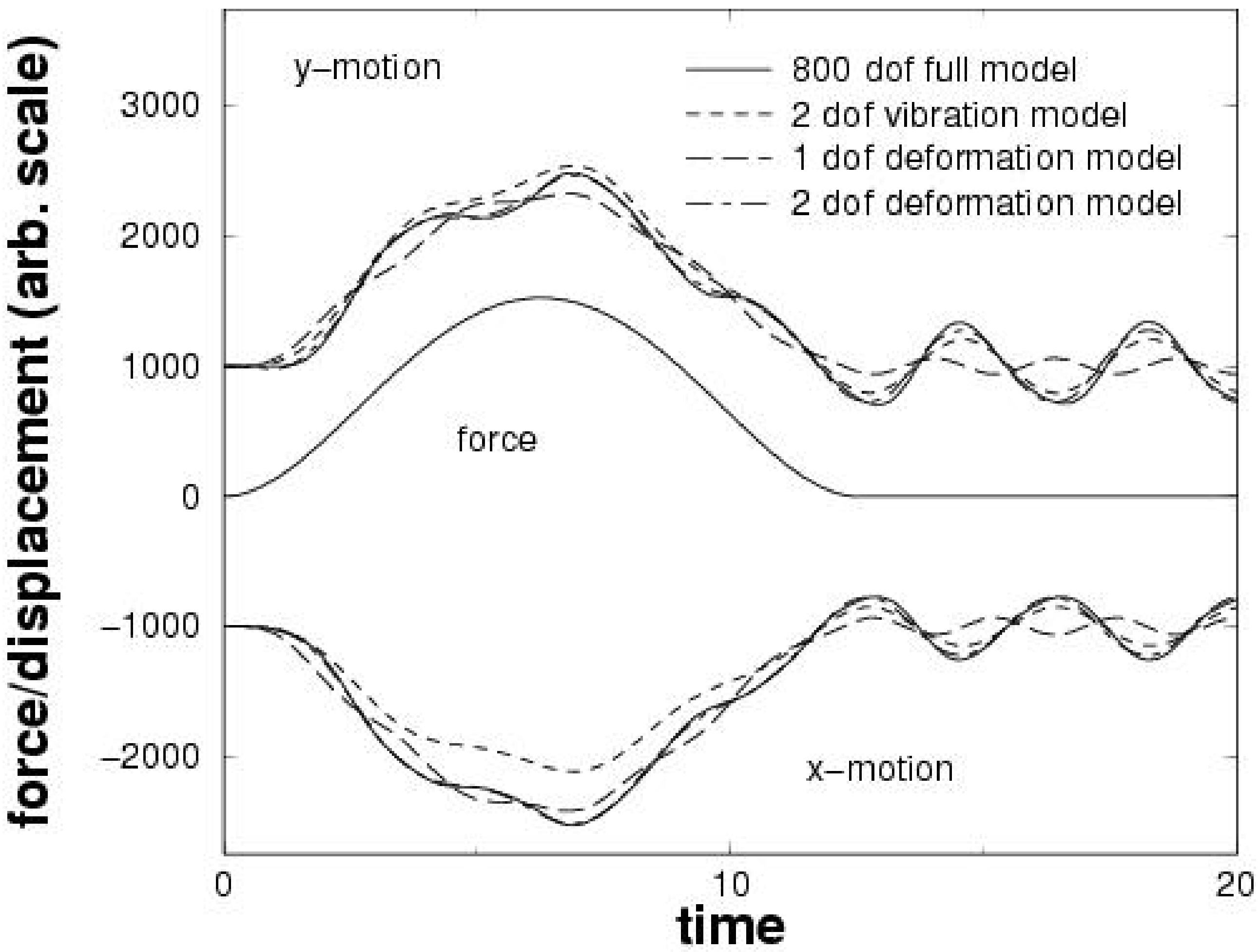
AN EXAMPLE





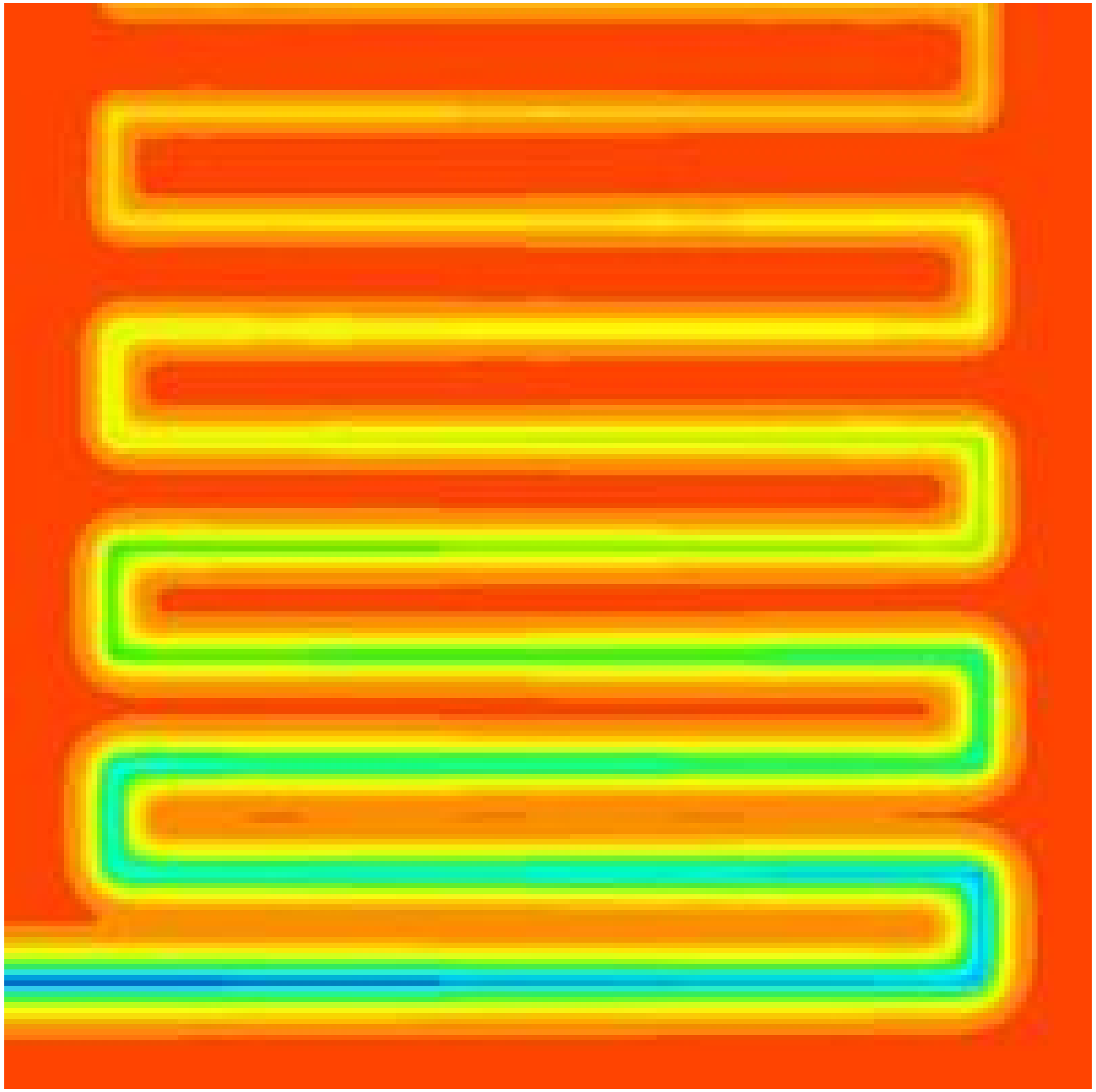
dim $X = 800$, dim $DX = Y = 2209$

Model reduction based on responses to $\mathbf{F}$ and $\ddot{\mathbf{F}}$



error in corner displacement of 2-dof model ~ 0.01

Cooling of a Plate, *as fast as possible*



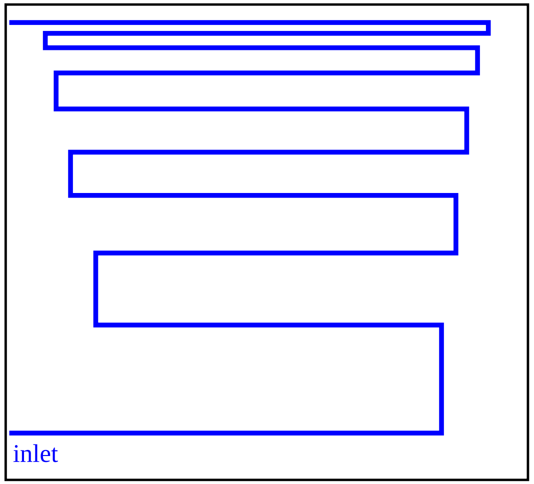
Constant flow heat-exchanger

$\Rightarrow$ ∞-long pipe

since $\dot{Q} = C(T_{\text{out}} - T_{\text{in}})$

Control: geometry of the pipe

Optimal design: constant exchange rate per volume



Control pairs (boundary or distributed)

effort $\otimes$ flow = power

force $\otimes$ displacement, voltage $\otimes$ current, $\dot{u} \otimes \partial_\nu f$, and $\dot{f} \otimes \partial_\nu u$, etc.

In partial integration (strong $\leftrightarrow$ weak):

$$-\int u \Delta v \rightarrow \int \nabla u \cdot \nabla v - \underbrace{\int_\partial u \partial_\nu v}_{\text{canceled?}}$$

boundary (divergence) terms are normally set to zero, but can be associated with boundary control.

Energy flux $\mathbf{S}$ is a pair of input and output

Given the Lagrangian $L(y, \dot{y})$, independent of t and x yields conserved energy E:

$$\dot{E} = \int \dot{H} - \int_{\partial} \mathbf{S} \cdot n$$

the energy flux $\mathbf{S}$ can be decomposed in: effort $\mathbf{e}$ and flow f, naturally the effort being vector-like. (However, $\mathbf{S} = \mathbf{f}e$ also possible.)

Generic differential equation in canonical pairs:

$$\begin{pmatrix} e_{\text{micro}}(x) \\ f_{\text{micro}}(x) \end{pmatrix} = \begin{pmatrix} 0 & \nabla\cdot \\ \nabla & 0 \end{pmatrix} \begin{pmatrix} e_{\text{macro}}(x) \\ f_{\text{macro}}(x) \end{pmatrix}$$

with the boundary variables $e_{\text{boundary}} = e_{\text{macro}}|_\partial$ and $f_{\text{boundary}} = f_{\text{macro}}|_\partial$. (1 input-1 output)

E.g. *wave equation*, *diffusion equation*, $\cdots$

Discretization on function space $X \otimes Y$:

X: macroscopic fields (0-forms) $\subset_d H^1$

$Y = \nabla X$ microscopic fields (1-forms) $\subset_d L^2$

Energy conservation requires: (state $q(x)$)

□ exact integration on $X \otimes Y$.

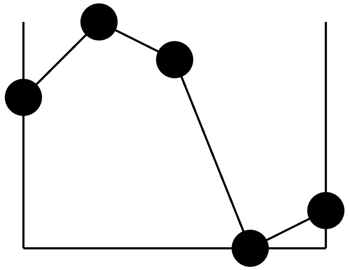
□ effective energies $\int dx H(q(x)) \to \sum V(q_h)$.

□ nonlinear: $\ddot{q}$ -or- $\dot{q} = f_1(\nabla \mathrm{f}_2(\nabla f_3(q, x), x), x)$.

results with stable nonlinear wave-equations (coll. LAGEP, Lyon)

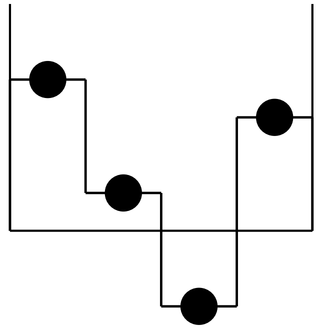
piece-wise
$\phi(z) = az + b$

piece-wise
$\psi(z) = a$



$\xrightarrow{D}$

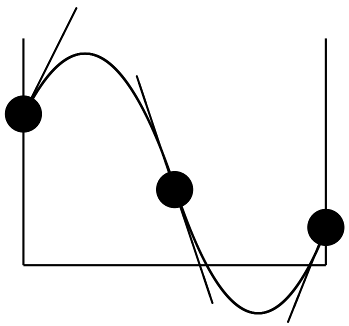
dim $X = 5$
dim $Y = 4$



Two pairs of local function spaces

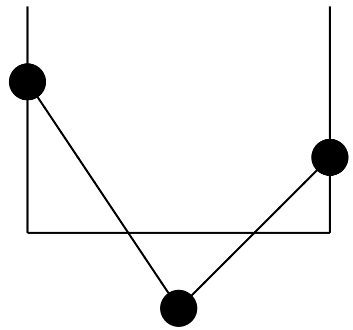
piece-wise
$\phi = az^2 + bz + c$

piece-wise
$\psi = 2az + b$



$\xrightarrow{D}$

dim $X = 6$
dim $Y = 3$

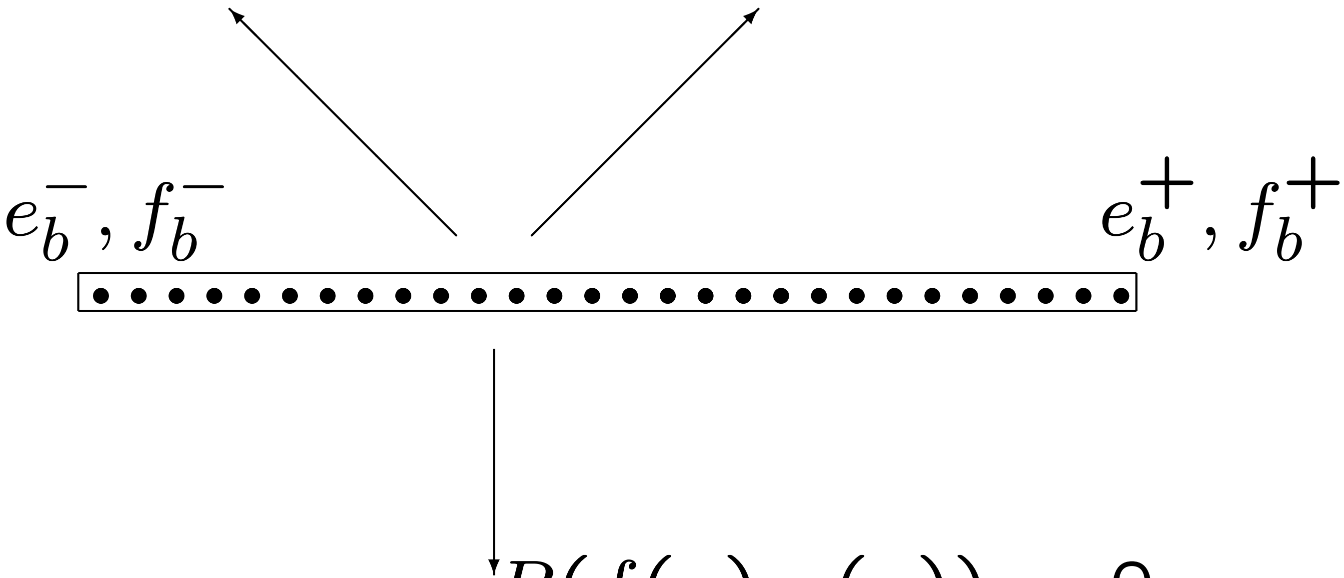


“**Systems of Conservation Laws**” (Lax)

$\rightarrow$ linear part

Nonlinear parts in energy storage and dissipation

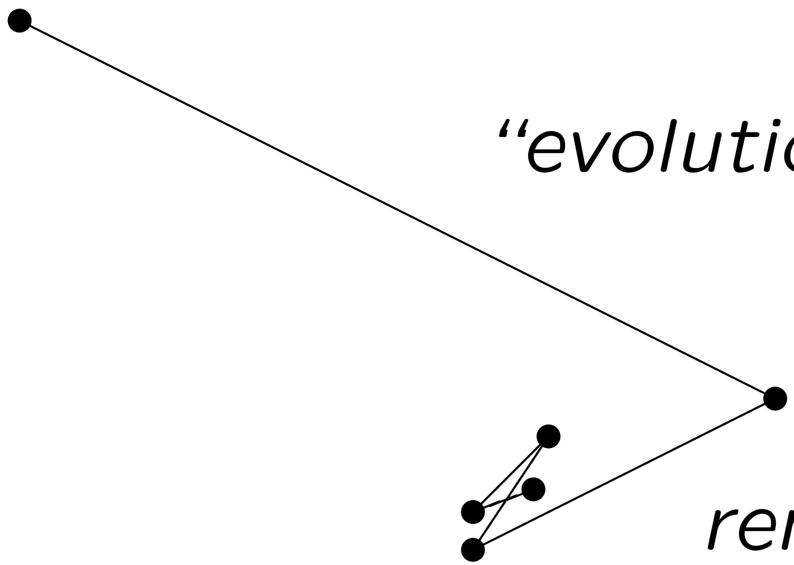
$$C(f(x), \dot{e}(x)) = 0 \qquad I(\dot{f}(x), e(x)) = 0$$



$$R(f(x), e(x)) = 0$$

Iteration $\rightarrow$ evolution-with-damping

The directional s: $\nabla_i \nabla_j J = \mathrm{s} \nabla J$
is incorporated in the second field.



“evolution by PID controller”

remember input-output

pick the right damping R, and step size dt

keywords: *Lyapunov, passitivity control*

Conclusions

□ Optimization usually splits into two parts: design and control

□ Input-output focus restricts dynamical state space

□ Effort-flow pairs and energy analysis yield natural boundary variables