



- \* **input-dependent "core model"**
- \* **core-model based reduction**
- \* **fluxes and differential operators**
- \* **simulation and initialization**
- \* **nonlinear dynamics**
- \* **writing and case studies**

MTNS contribution: (July 2006, Kyoto, Japan)

Physical model reduction of interacting, continuous systems

Symplectic story:

$$\begin{array}{c} \bullet \\ \mathbf{x} \\ \bullet \\ \mathbf{p} \end{array} = \begin{array}{|c|c|} \hline 0 & \mathbf{I} \\ \hline -\mathbf{I} & 0 \\ \hline \end{array} \begin{array}{c} d_{\mathbf{x}} H \\ d_{\mathbf{p}} H \end{array} \xrightarrow{\text{flux}} \begin{array}{c} \bullet \\ \mathbf{q} \\ \bullet \\ \mathbf{p} \end{array} = \begin{array}{|c|c|} \hline 0 & \mathbf{D} \\ \hline -\mathbf{D}^* & 0 \\ \hline \end{array} \begin{array}{c} d_{\mathbf{q}} H \\ d_{\mathbf{p}} H \end{array}$$

isolated Hamiltonian

port-Hamiltonian

$$\dot{\mathbf{g}} = \nabla \cdot \mathbf{J}$$

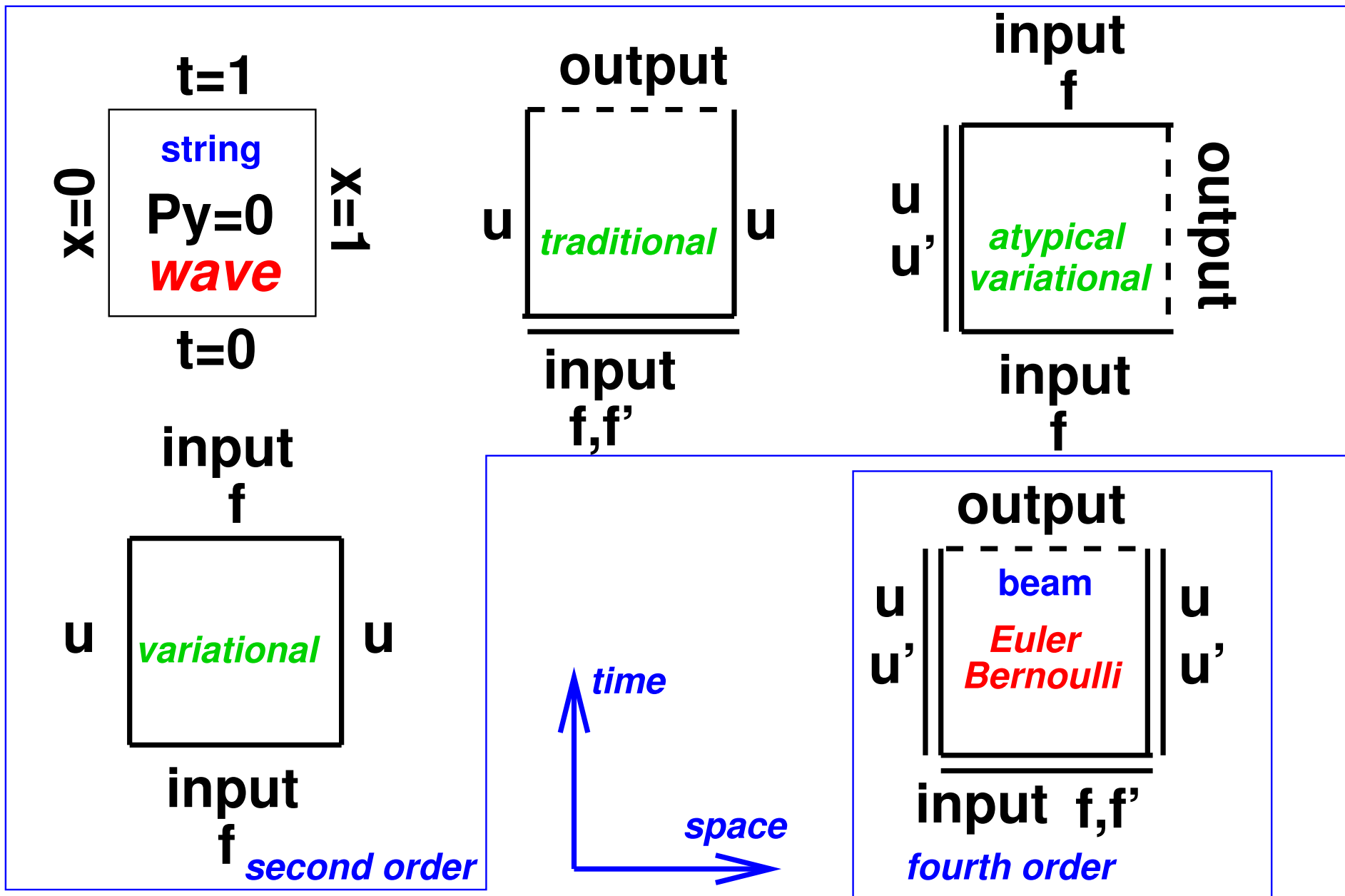
$$\begin{array}{c} \bullet \\ \mathbf{Q} \\ \bullet \\ \mathbf{P} \end{array} \xrightarrow{\text{null space}} = \begin{array}{|c|c|} \hline 0 & \mathbf{S}_+ \\ \hline \mathbf{S}_- & 0 \\ \hline \end{array} \begin{array}{c} d_{\mathbf{Q}} H \\ d_{\mathbf{P}} H \end{array}$$

discrete port-Hamiltonian

S: half-infinite shift operators

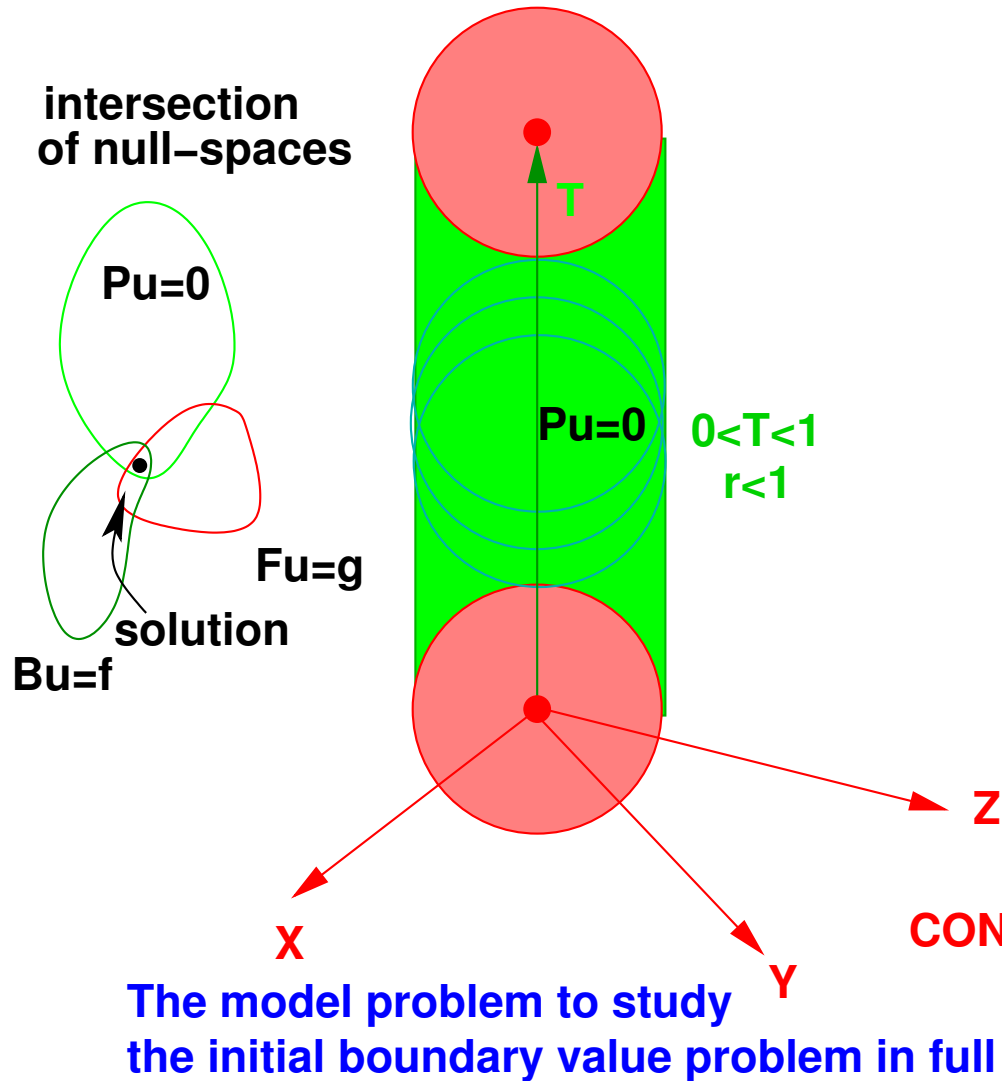
$$\mathbf{S} = \begin{array}{|c|c|c|c|c|} \hline 0 & 1 & & & \\ \hline \text{null-space} & 0 & 1 & & \\ \hline & & 0 & 1 & \\ \hline & & & 0 & 1 \\ \hline & & & & 0 \\ \hline \end{array}$$

# INITIAL BOUNDARY VALUE PROBLEM

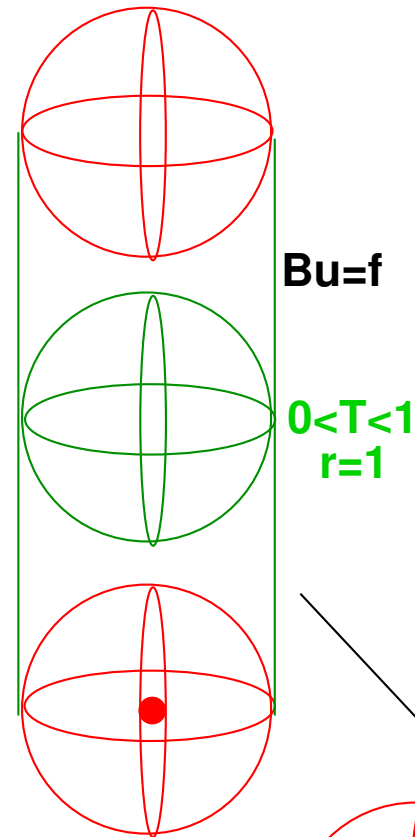


# BALL CONTROL

## FULL PROBLEM

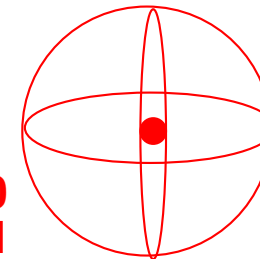


## BOUNDARY CONDITIONS

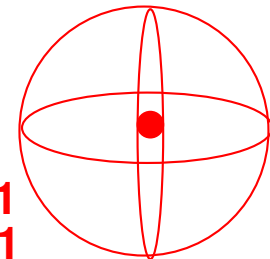


## CONSISTENCY

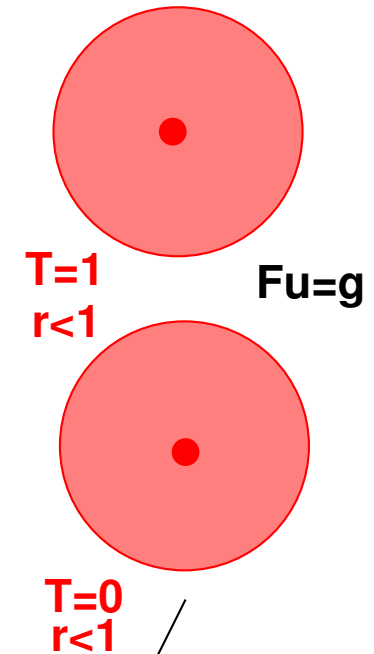
$T=0$   
 $r=1$



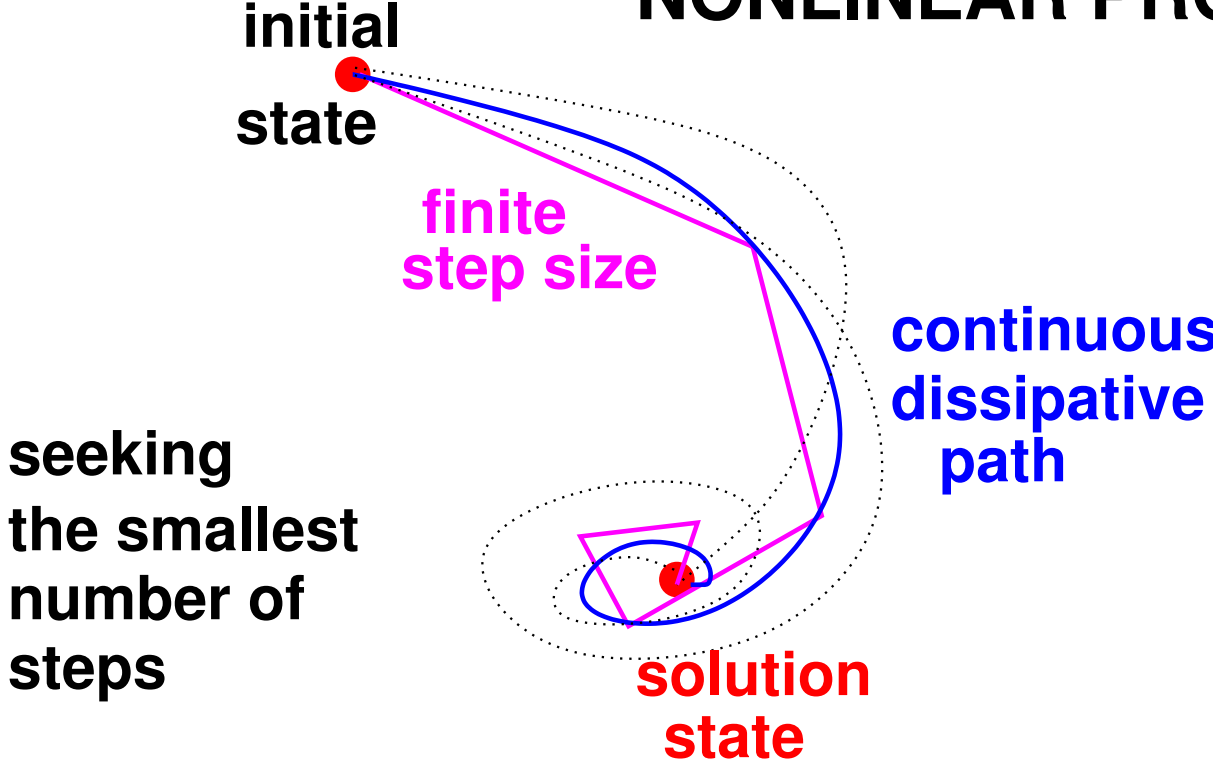
$T=1$   
 $r=1$



## INITIAL/FINAL CONDITIONS

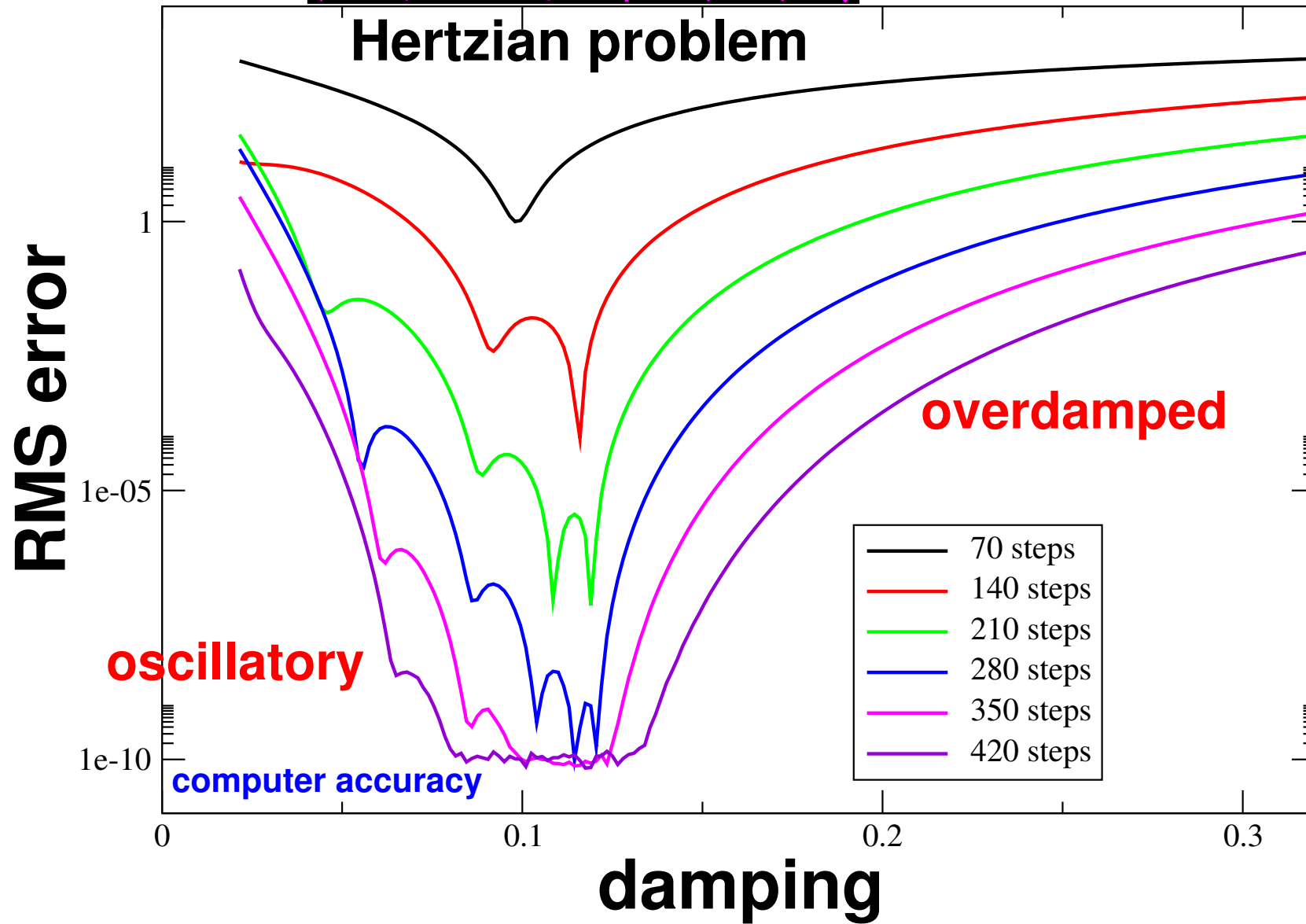
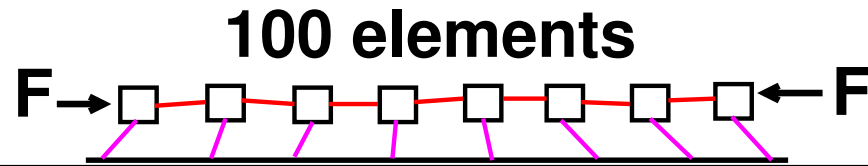


# ITERATIVE METHODS FOR LINEAR AND NONLINEAR PROBLEMS



**step size limited by  $\|A\|$**   
(improvements by integration methods)

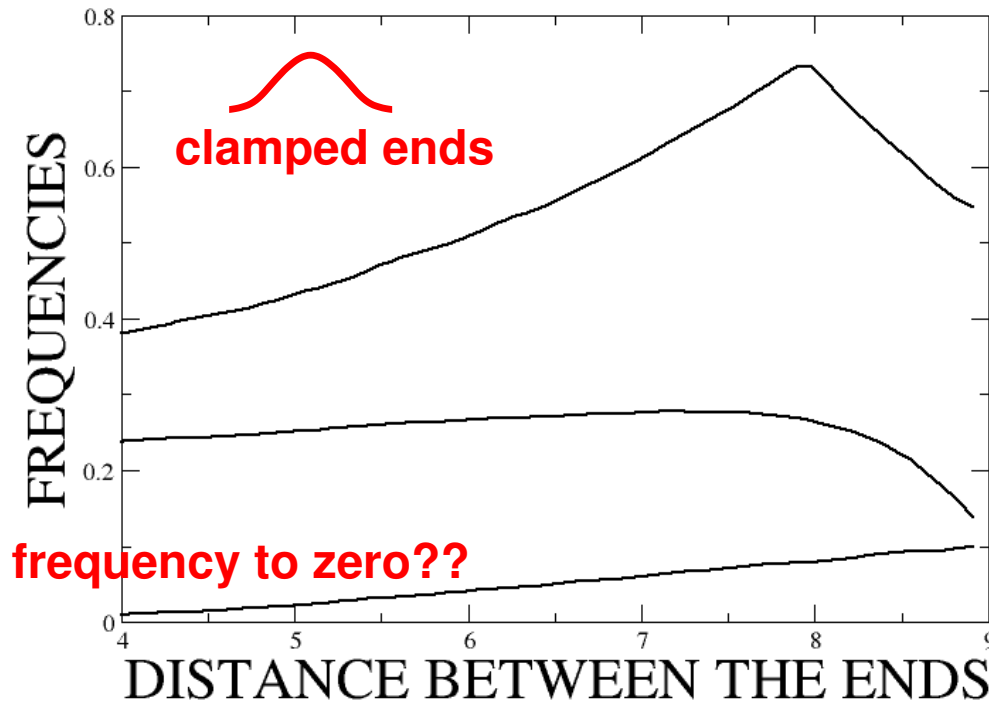
**optimal dissipation:**  
"not too small, not too large"



# Multiple solutions for large deformations

## CLAMPED BEAM

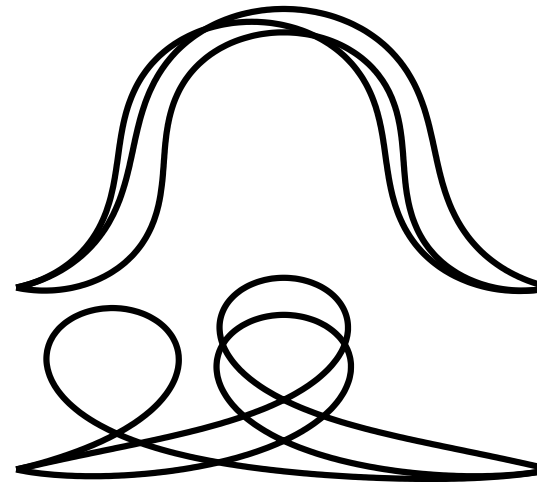
STRAIGHT CLAMPED ENDS



## "Swagger signatures"

- \*cross-over solutions
- \*flat energy minima
- \*strong bc dependence

solutions for the same boundary conditions

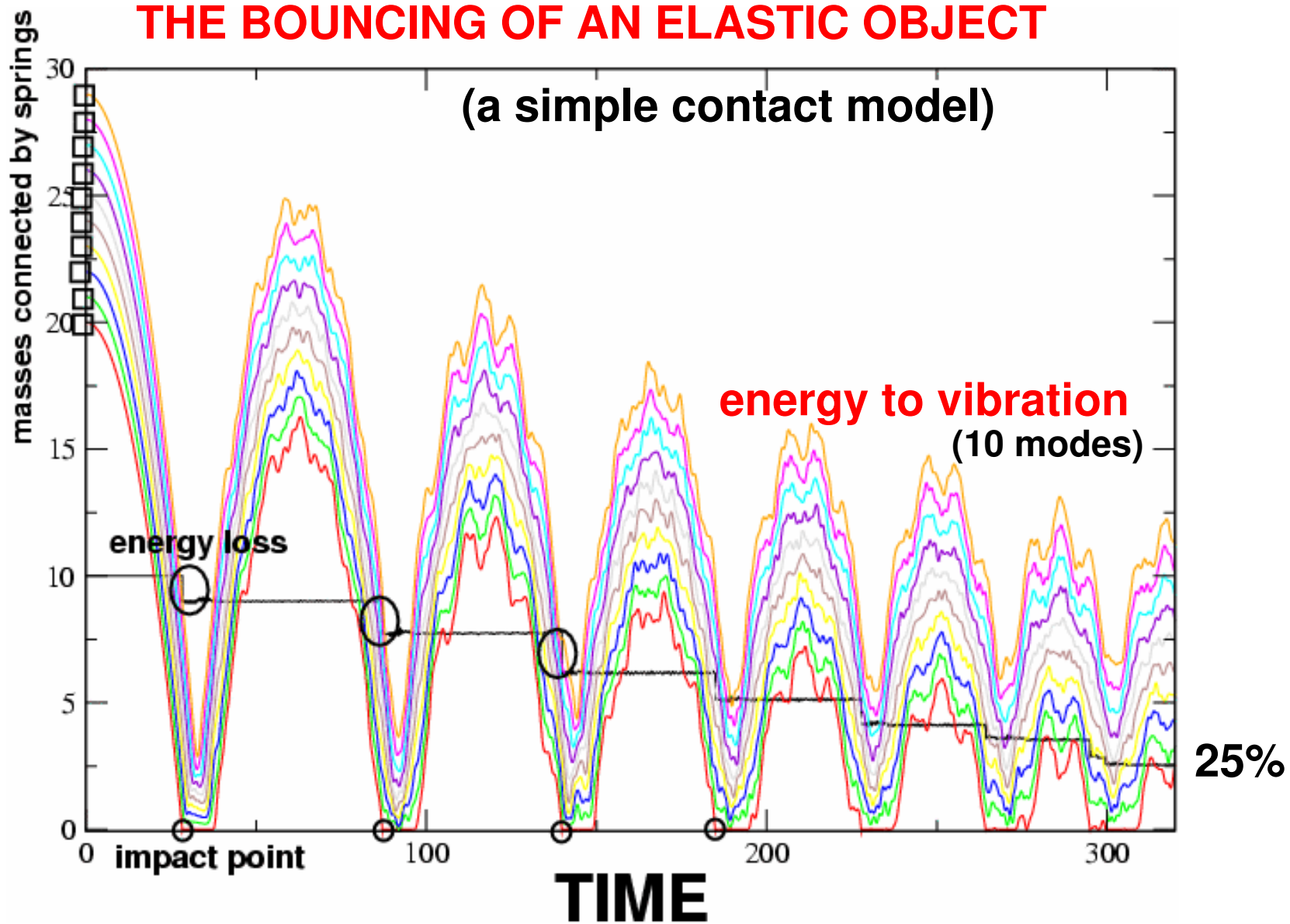


large changes in configuration for small changes in boundary conditions

(related to bifurcation theory of Antman et al.)

# THE BOUNCING OF AN ELASTIC OBJECT

(a simple contact model)



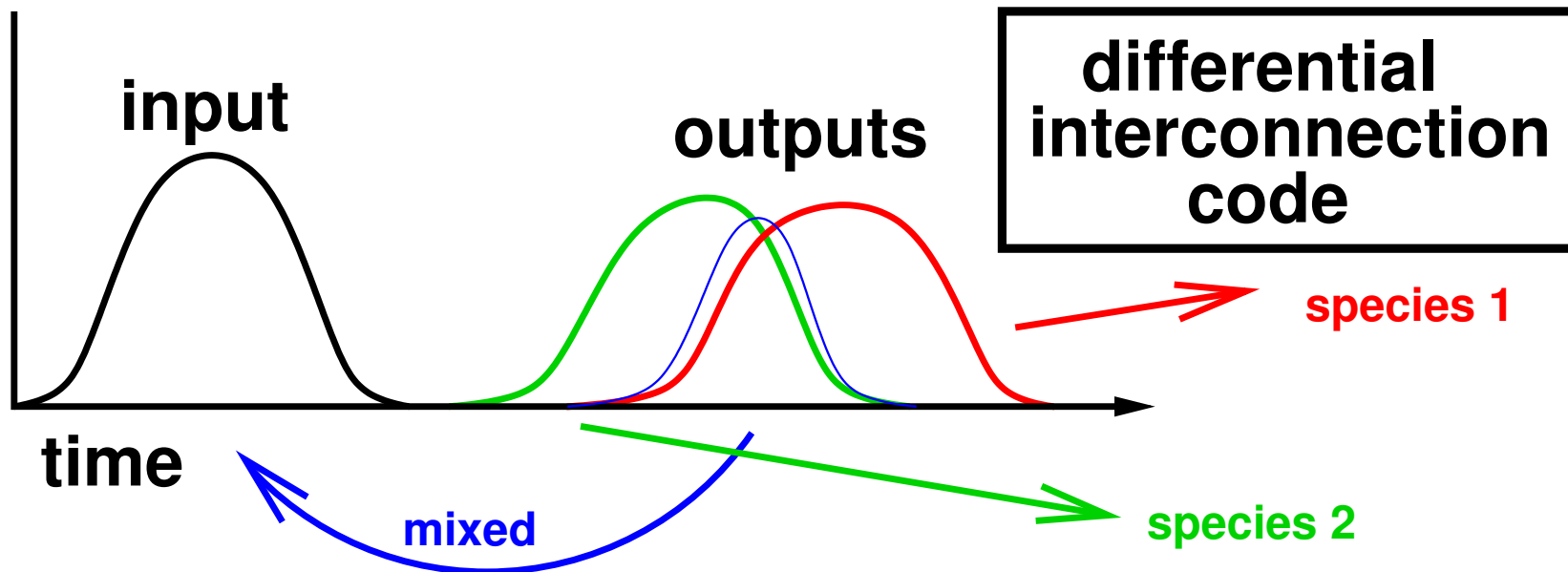
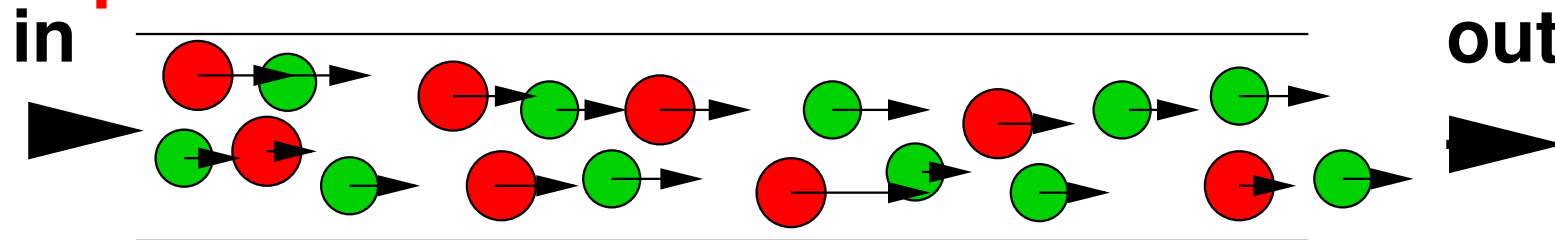


# Two-species Stefan–Maxwell diffusion

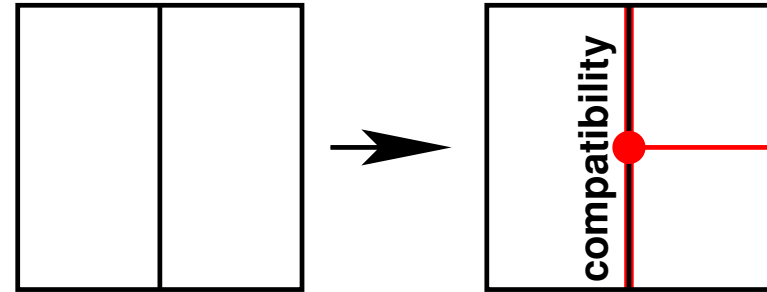
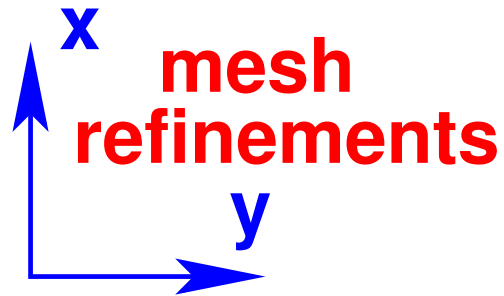
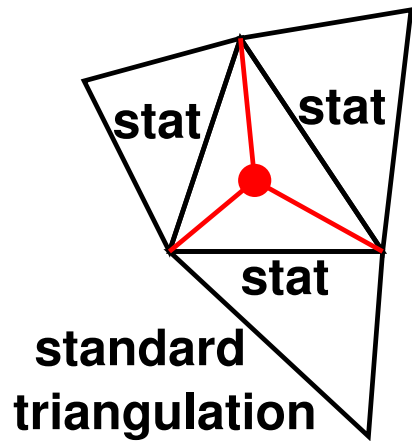
(separation technique)

diffusion differences  
mutual dependence  
nonlinear theory  
quasi 3D zeolite

(In collaboration with Lefevre and Couenne)



# PaLIS (point and line interpolated surfaces)

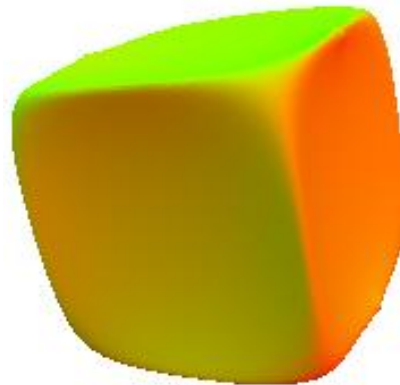
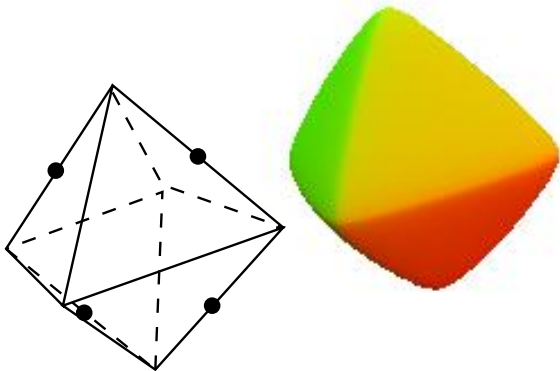


Physical principles, with arbitrary smoothness

small rigidity

large rigidity

"natural" rigidity



modular approach

FILE specs

separate stages data in and out

PDE types

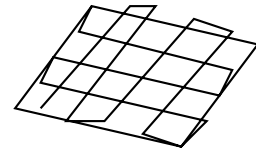
Causality problem

→



→

MESH data



FE/modes/splines

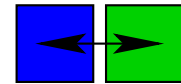
PARAMETER constants

→



→

NETWORK data



Hamiltonian generation

CONSTRAINT fixed input

→



→

HAMILTONIAN data

initialization

H=...  
dH=...

PORT data

→



→

STATE data

x=...

simulation/interaction

co-simulation

I/O data

y=...

GRAPHICS data

x11, opengl, ..

(student-dll-20sim testing (Eric Staal))



# **OUTLOOK**

- \* automating Hamiltonian generation**
- \* nonlinear simulation, time integration**
- \* mathematical initial boundary value problem**
- \* case studies (Philips? Reden? Control problems?)**
- \* 20sim integration, simulation software**
- \* writing papers and thesis**