MODEL REDUCTION and PORT CONSTRUCTION

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*Model reduction (classic, $K=\infty$, $M=0$)

*Modes, diagonal versus dynamical

*Retaining Ports under Model Reduction

*Nonlinear aspects of Reduction

STW EWI/UT

PACDAS Infinite Control

OUTLINE

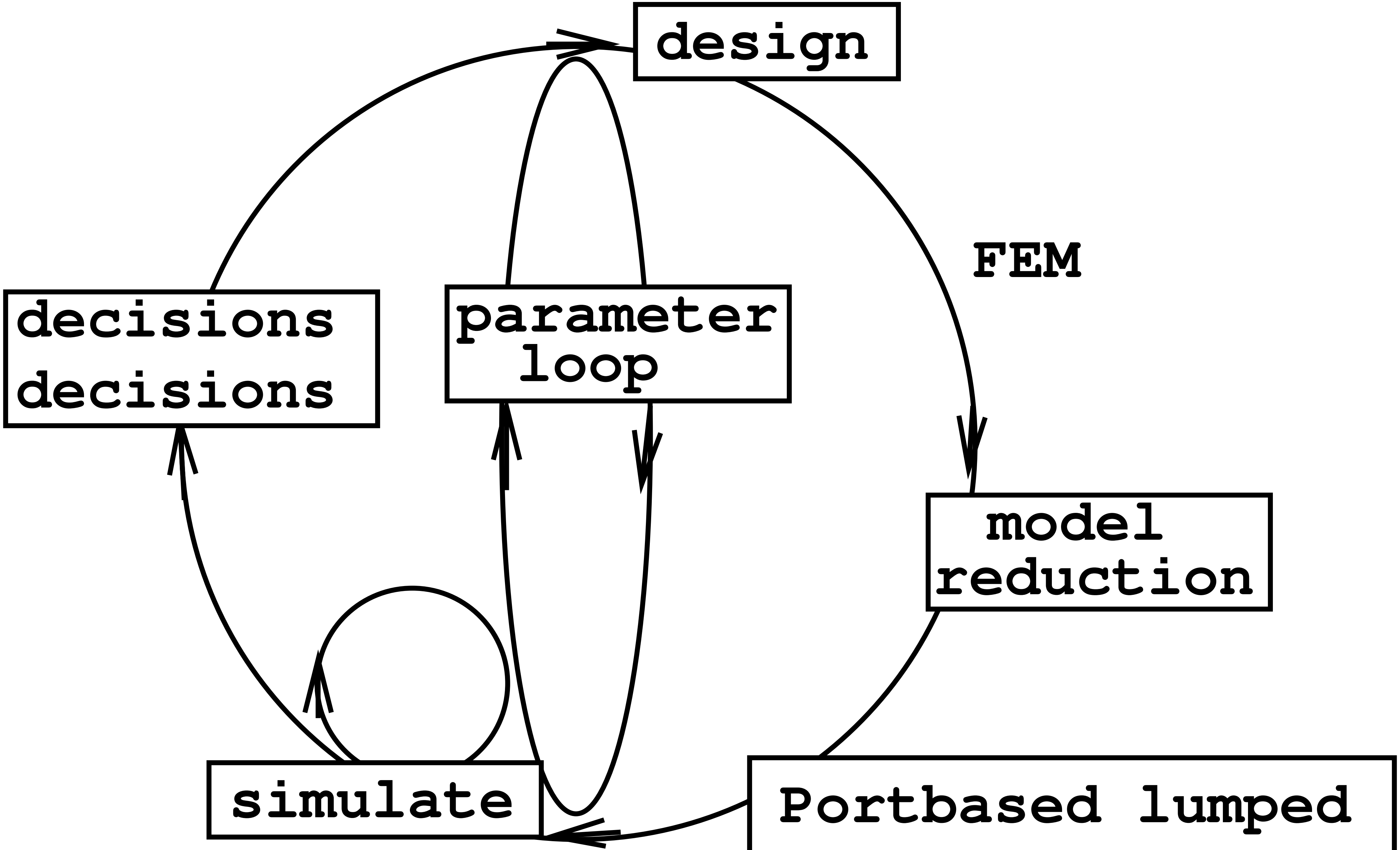
*design and simulation

*modal analysis

*model reduction

*nonlinearity

*numerical port issues


design
FEM
decisions
decisions
parameter
loop
model
reduction
simulate
Portbased lumped

retention

port
parameter
dynamics

← design concepts

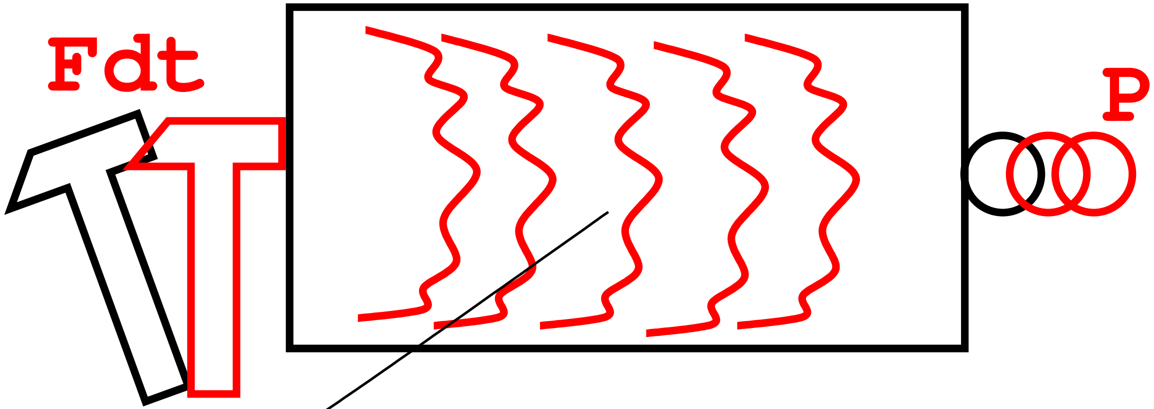
model reduction →

model freedom

port matching
state dimension
control
parameter values

← variable during simulation

KEEPING TRACK OF EFFORT AND FLOW



Bulk dynamics
+
Contact dynamics

Energy!

Momentum? (e.g. reflections)

Force?

LINEAR THEORY

$$M.x'' = -K.x - R.x'$$

mass — stiffness — damping

Lagrangean

$$L = x'.M.x'/2 - x.K.x/2$$

Hamiltonian

$$H = p.M^{-1}.p/2 + x.K.x/2$$

Damping (Rayleigh & Gossick)

$$F = x'.R.x'/2 + x''.G.x''/2$$

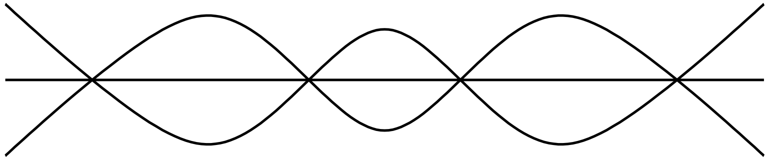
mode ϕ, frequency w
damping γ (as perturbation)

PROBLEMS WITH MODES

boundaries $\longrightarrow$ Craig-Bampton, etc

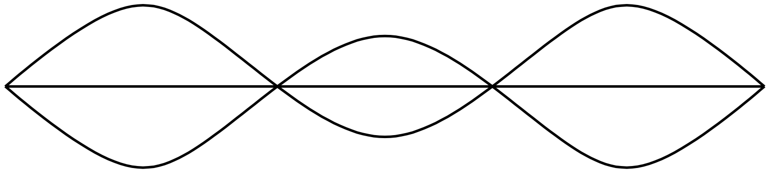
vibrations and .... transfer

VIBRATIONAL MODES; no transfer



free ends, $f=0$

$$P = f.dx' = 0$$

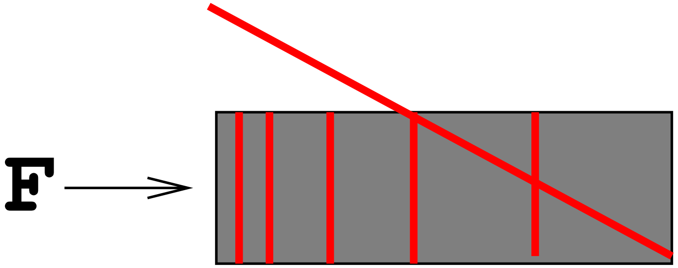


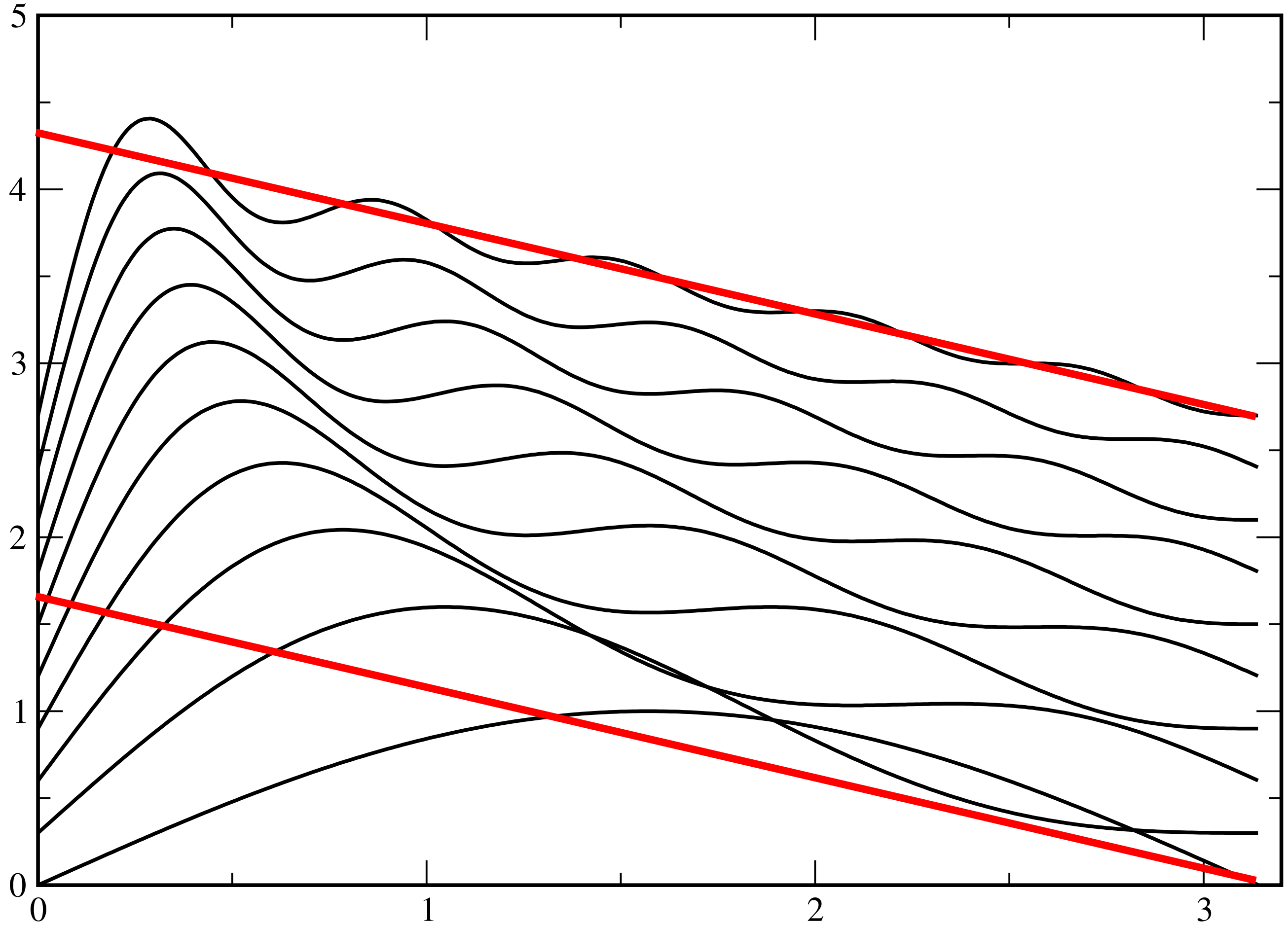
fixed ends, $x=0$

$$P = f.dx' = 0$$

$\longrightarrow$ **NO CONTROL**

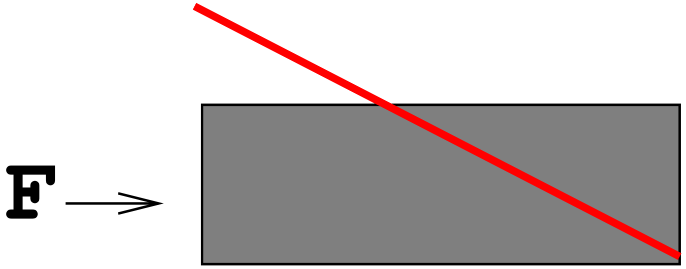
MODE EXPANSION, fixed ends

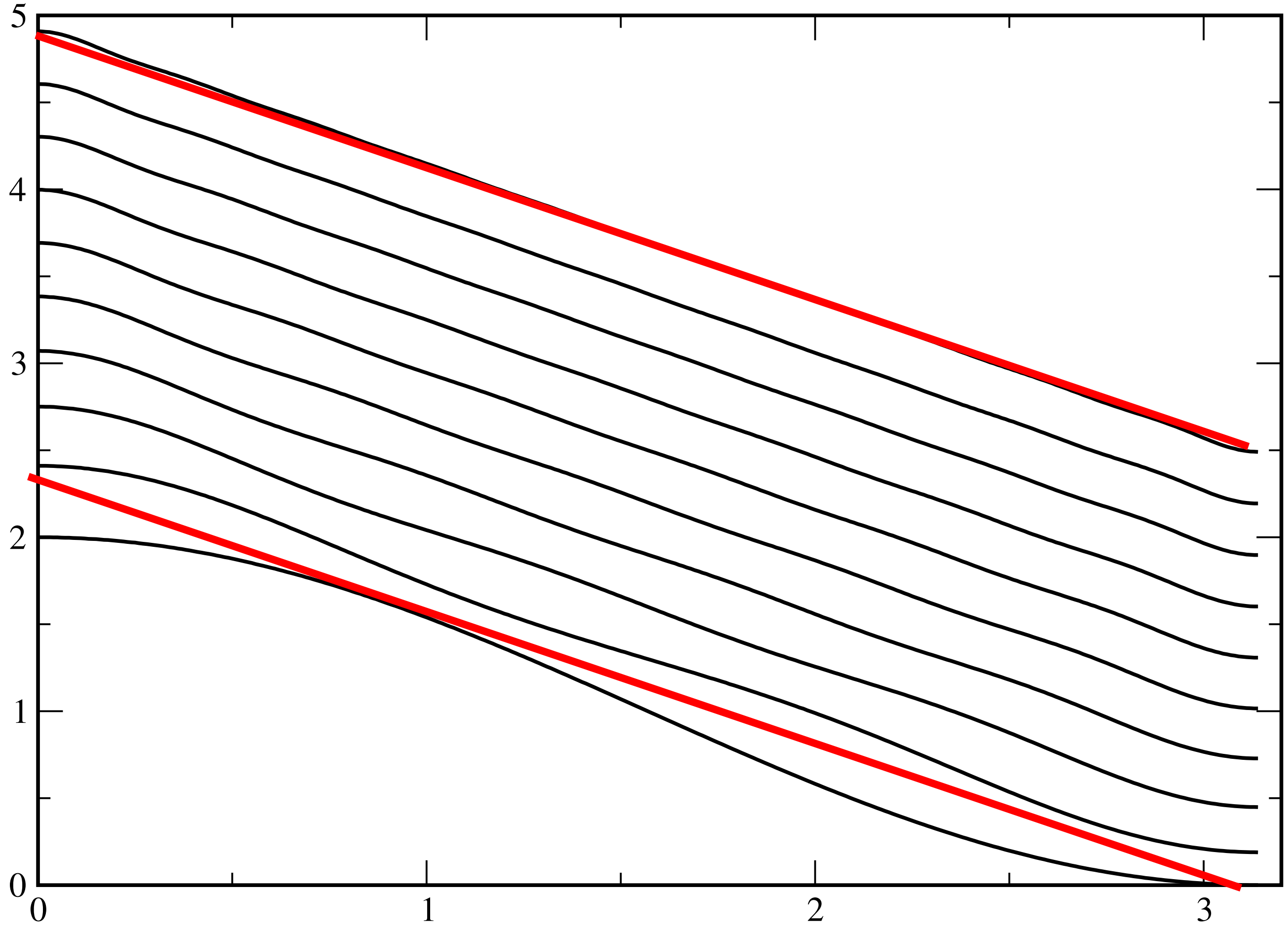




Poor strain fit, terrible end behaviour

MODE EXPANSION, free ends

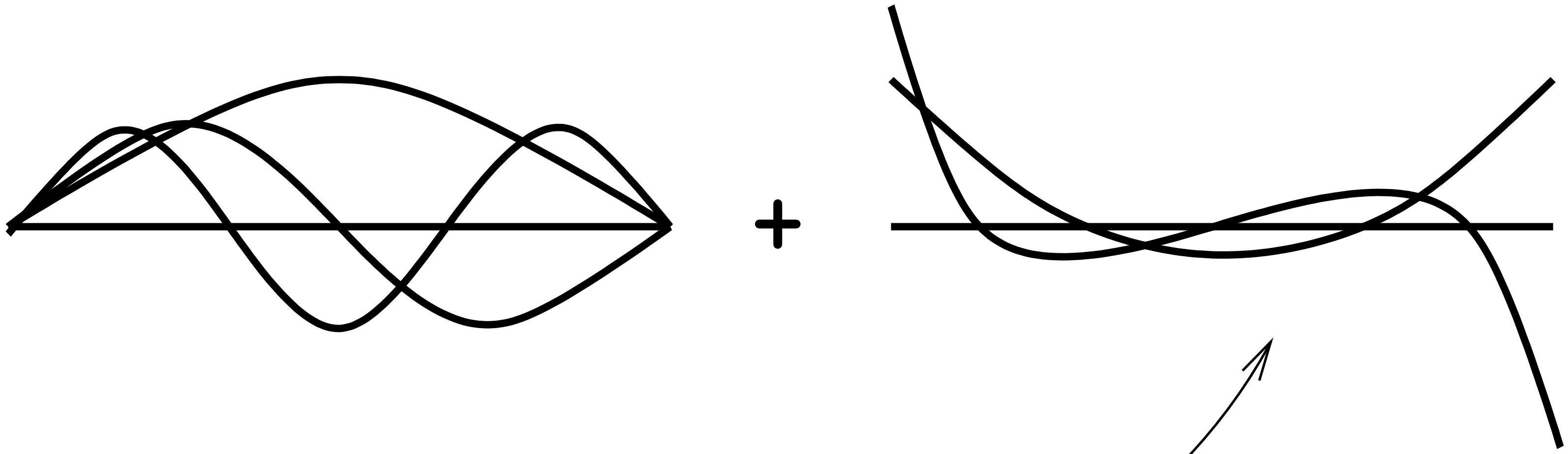




Better strain fit, but still poor end behaviour

CRAIG–BAMPTON (WANG, MARGOLIS, etc.)

repair job: adding "boundary modes"



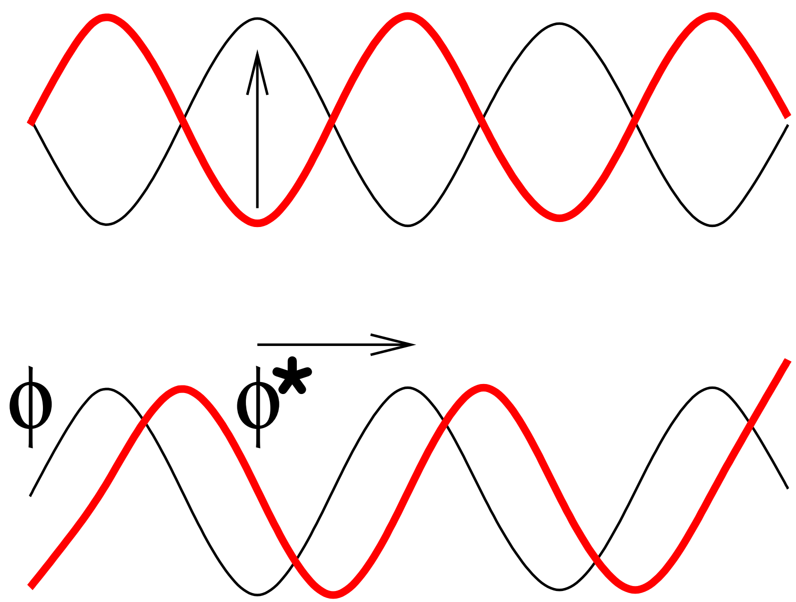
orthogonal

not orthogonal = transfer!

vibrational analysis: **boundary modes to pump energy into the system**

good for loosely connected systems

STANDING WAVES are easy,
MOVING WAVES are more difficult



$\sin(wt)\,\phi(x)$

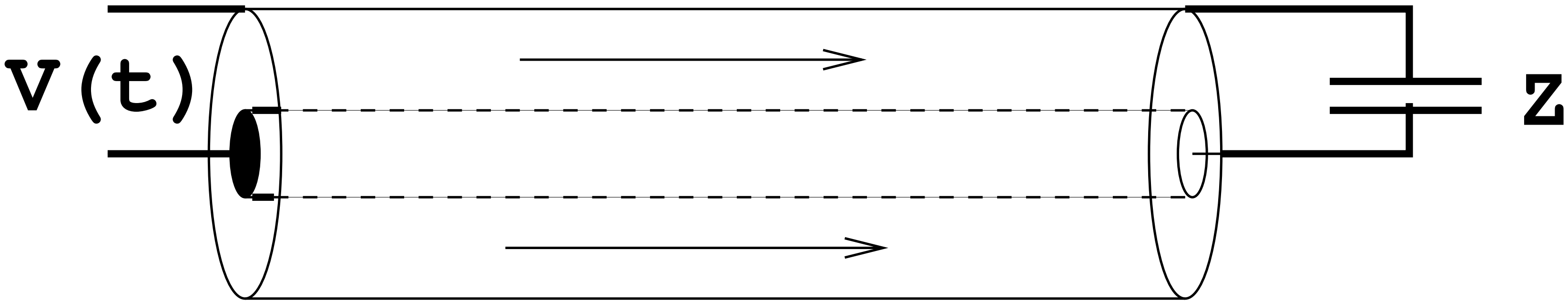
$\phi(x) \xrightarrow{t} \phi^*(x)$

in other words:

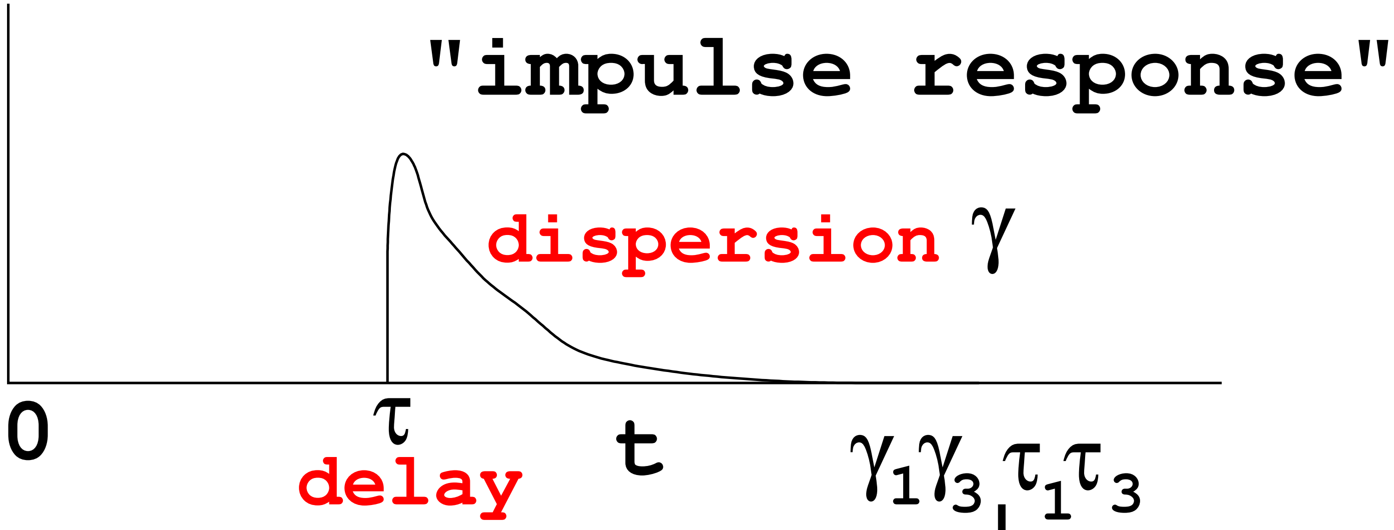
vibrational analysis is easy,

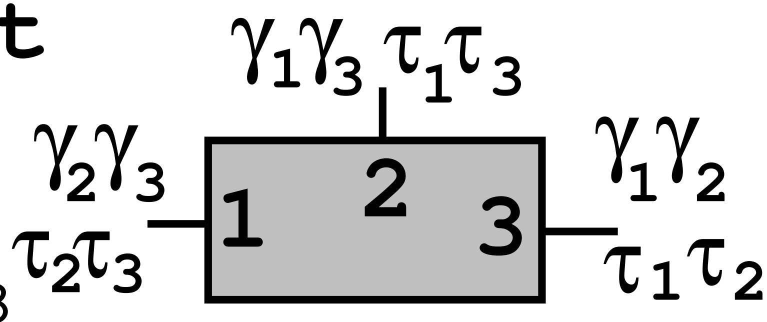
signal transfer not

TRANSFER MATRICES, minimal dynamics



end stop
complete
absorption



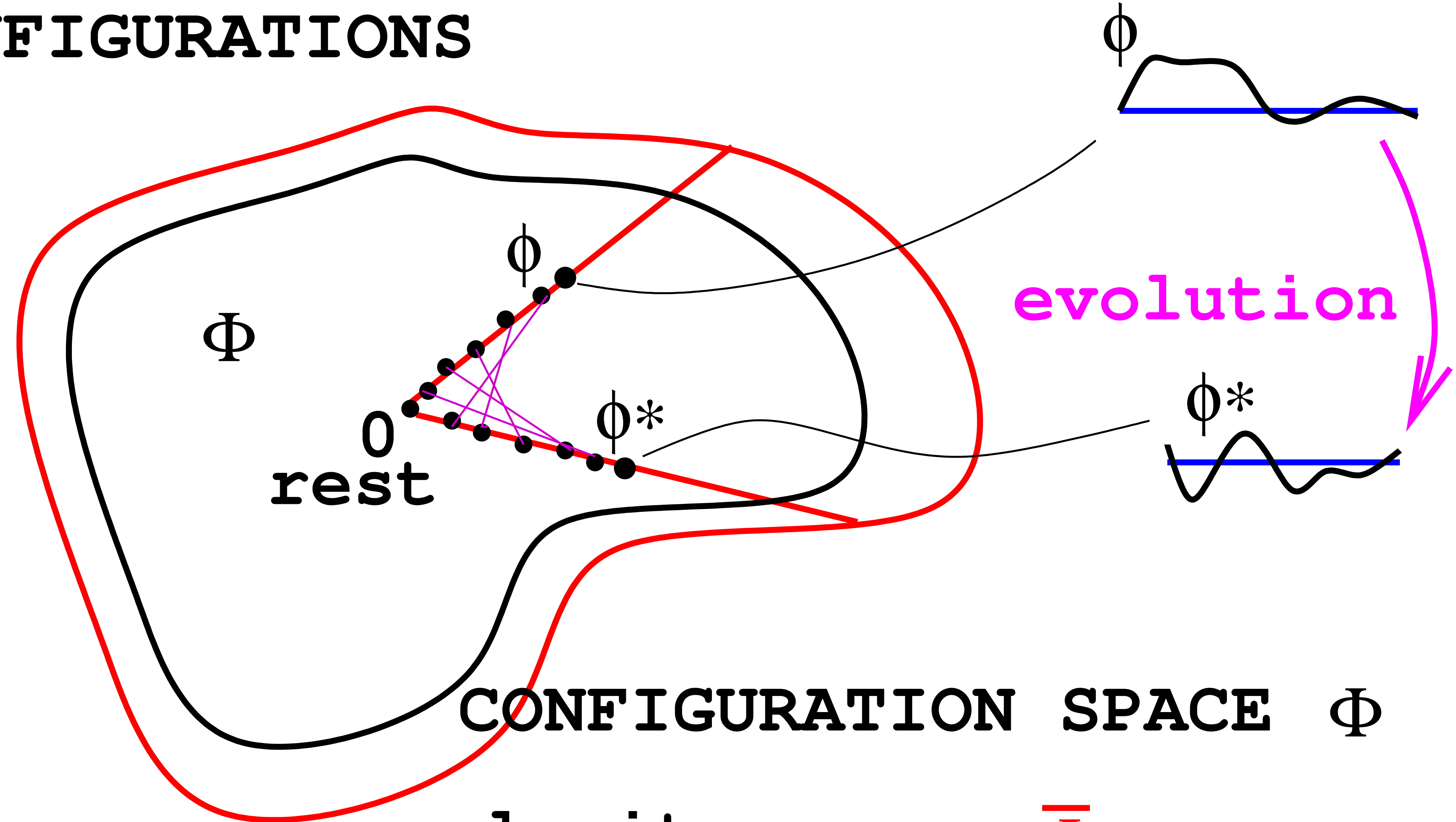


time scale
of impulse
response is
not typical

MODEL REDUCTION

concepts and examples

CONFIGURATIONS

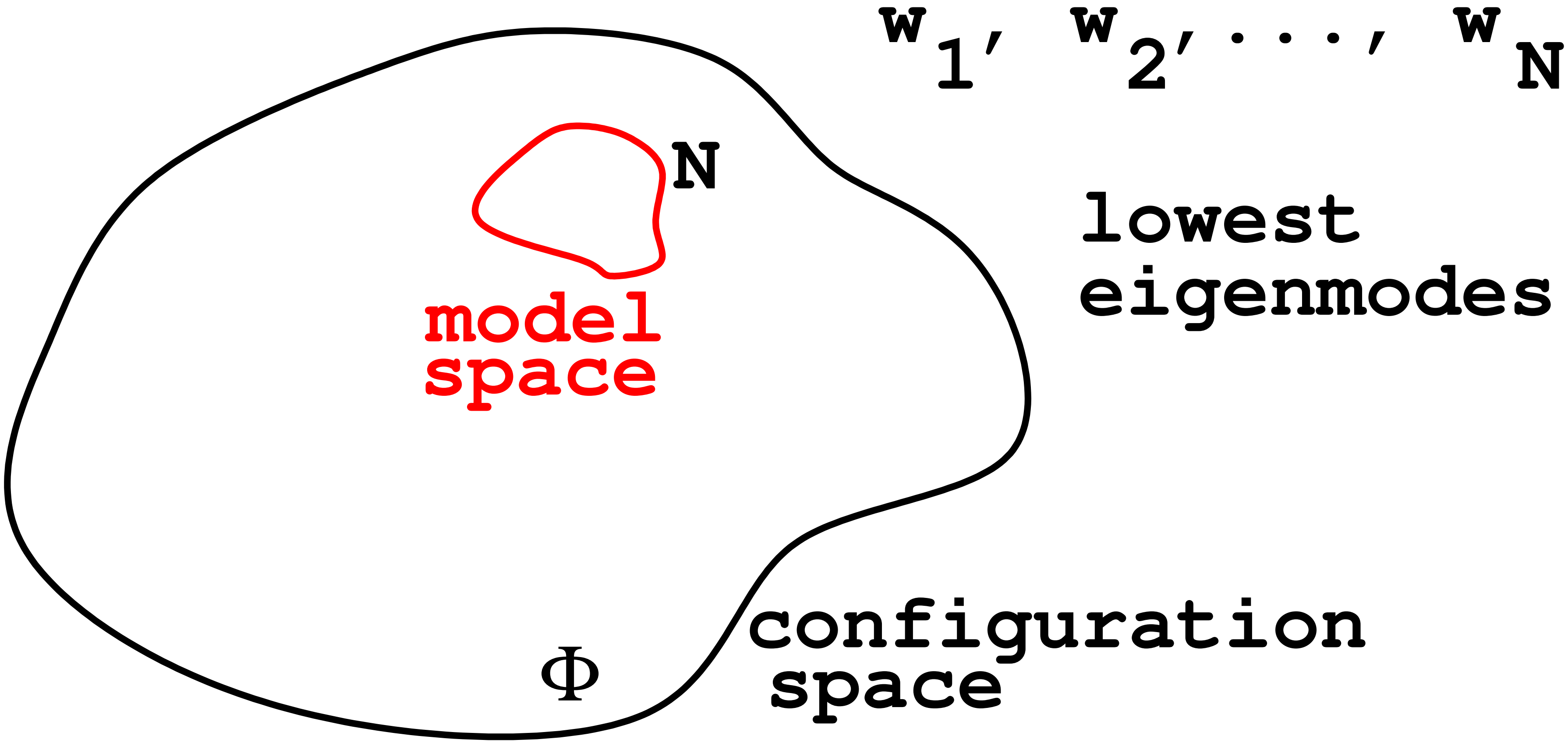


CONFIGURATION SPACE Φ

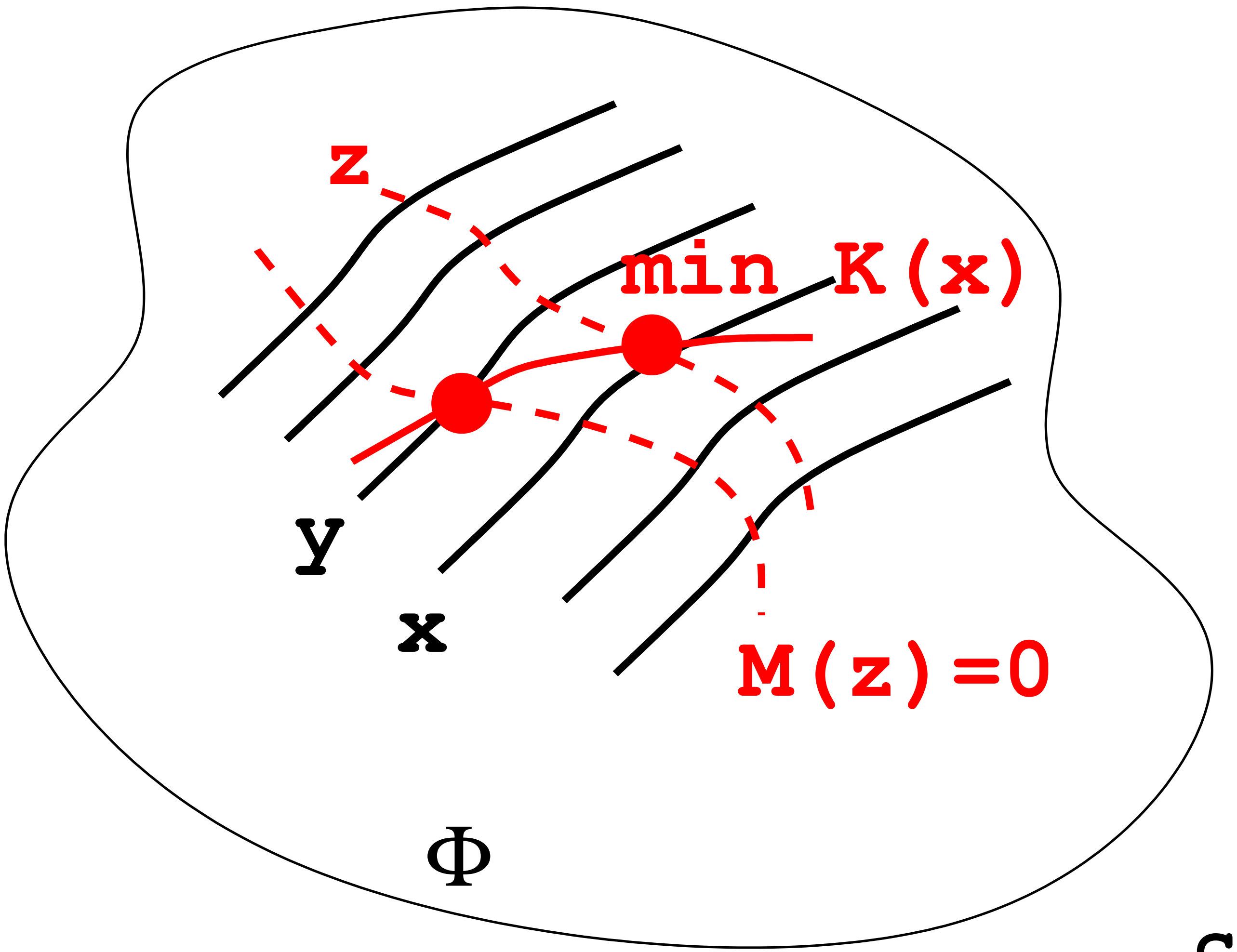
velocity space $\overline{\Phi}$

if Φ is linear, $\Phi \geqq \overline{\Phi}$

STANDARD MODEL REDUCTION



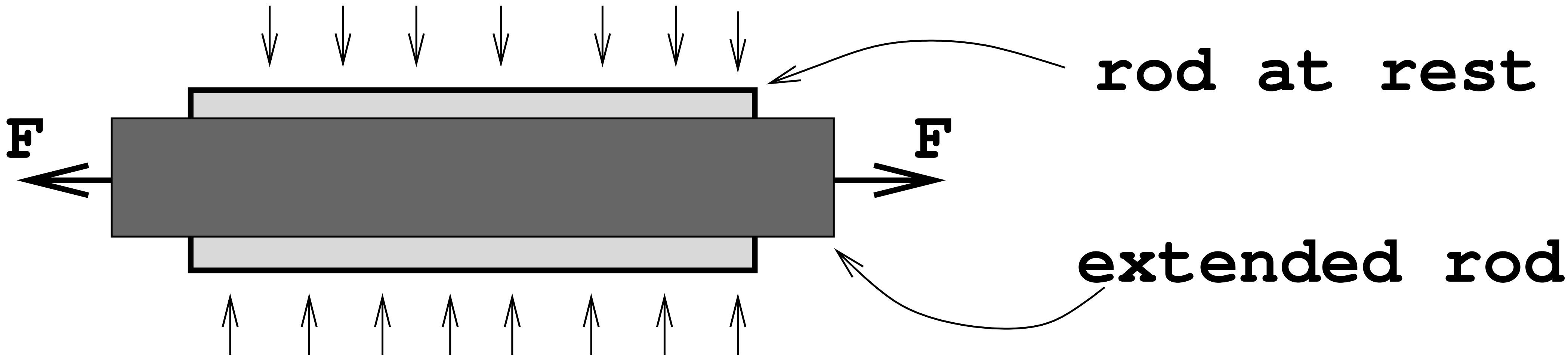
MODEL REDUCTION (M=0), no dynamics



optimal
configuration

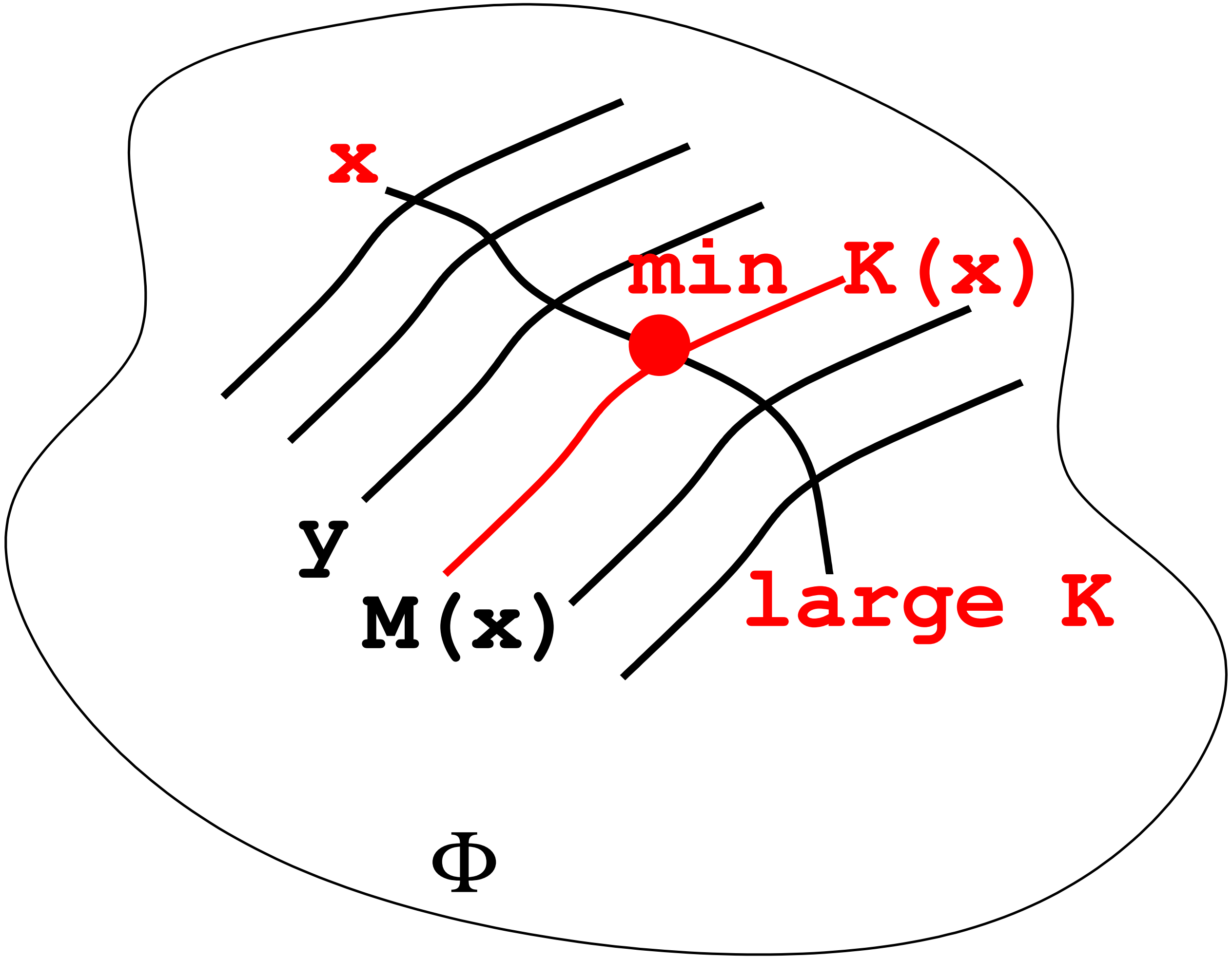
configuration space

POISSON RATIO, a case of M=0



A change in configuration, which reduces the elastic energy, with neglible kinetic energy

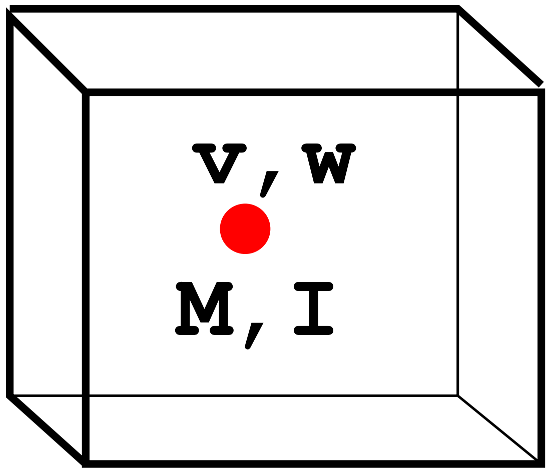
MODEL REDUCTION ($K=\infty$), no dynamics

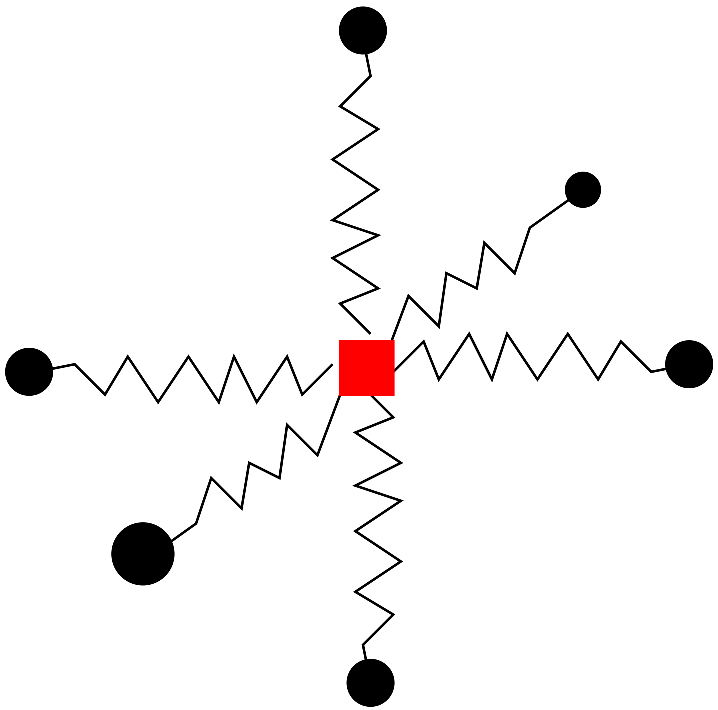


configuration space

rigid configuration

Centre-of-Mass coordinates and inertia

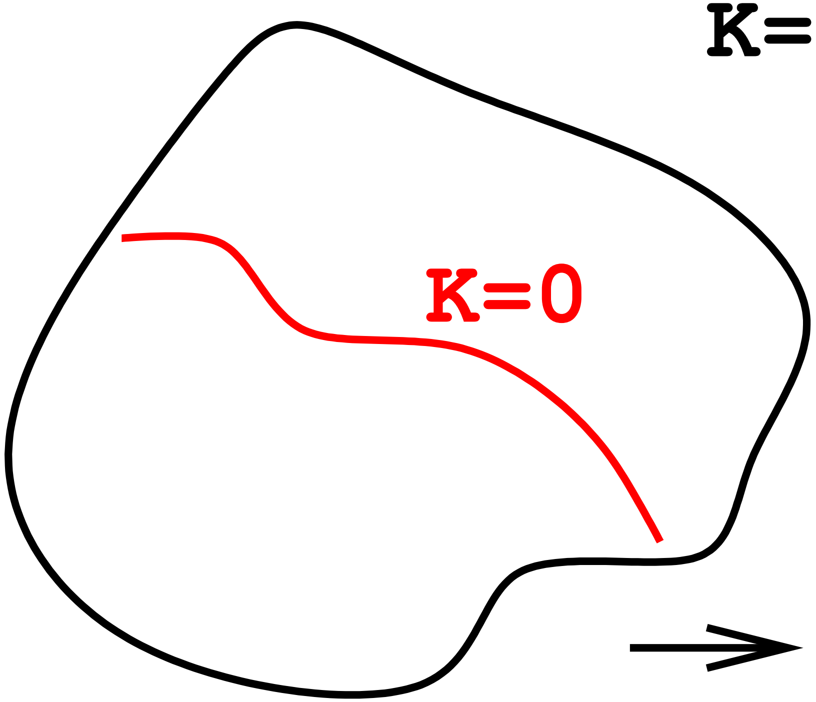


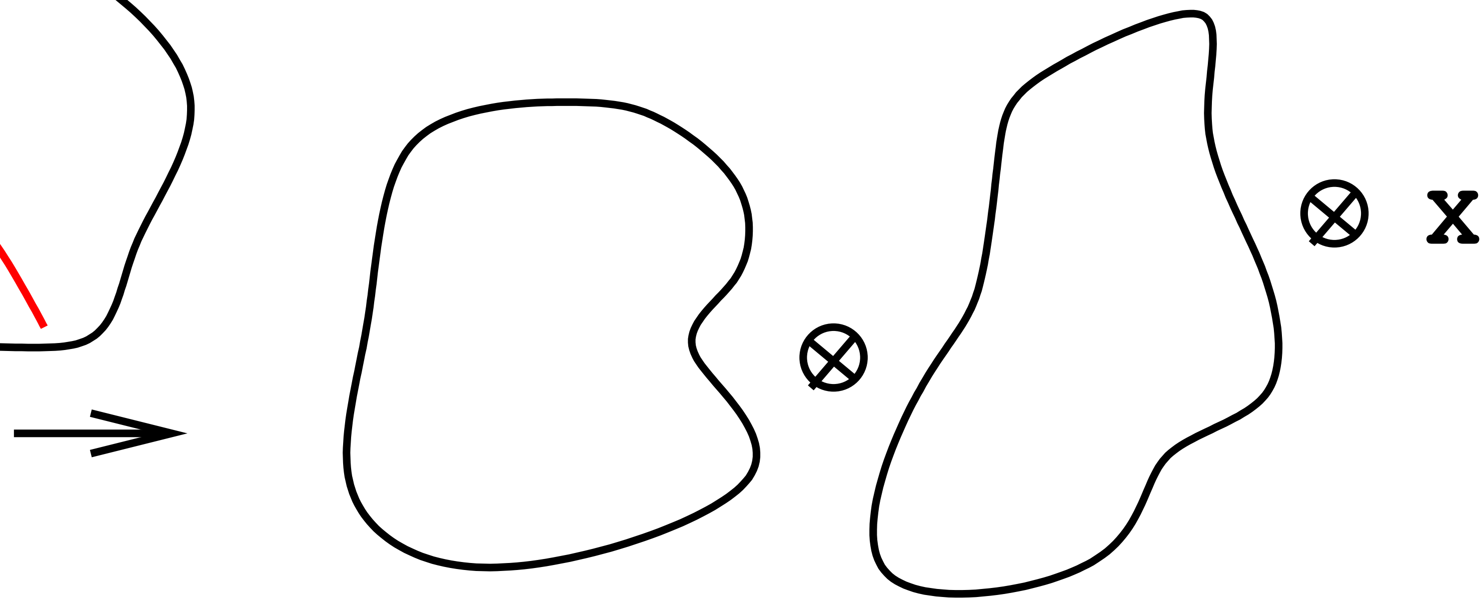


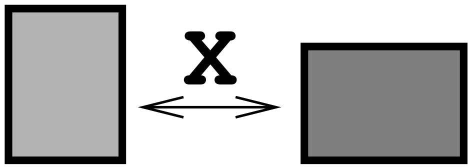
"Deformation does not affect the kinetic energies"

MODEL REDUCTION (K=0)

K=0: conserved momentum







free motion
or rotation

NONLINEARITY

microscopic or through reduction

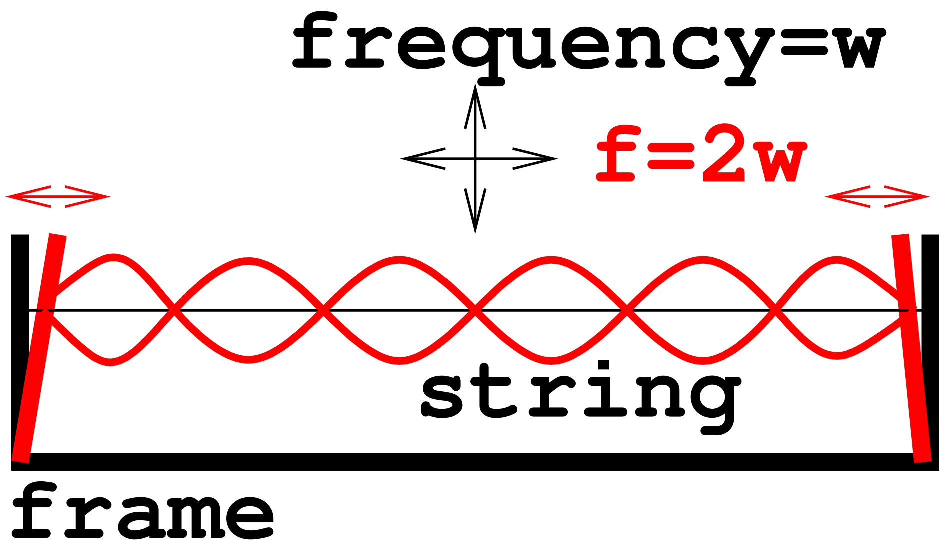
NONLINEAR ASPECTS OF REDUCTION

*body-frame coordinates (angles)

*perpendicular oscillations

$\longrightarrow$ two aspects of non-trivial configuration

2-dimensional motion of 1-dimensional object



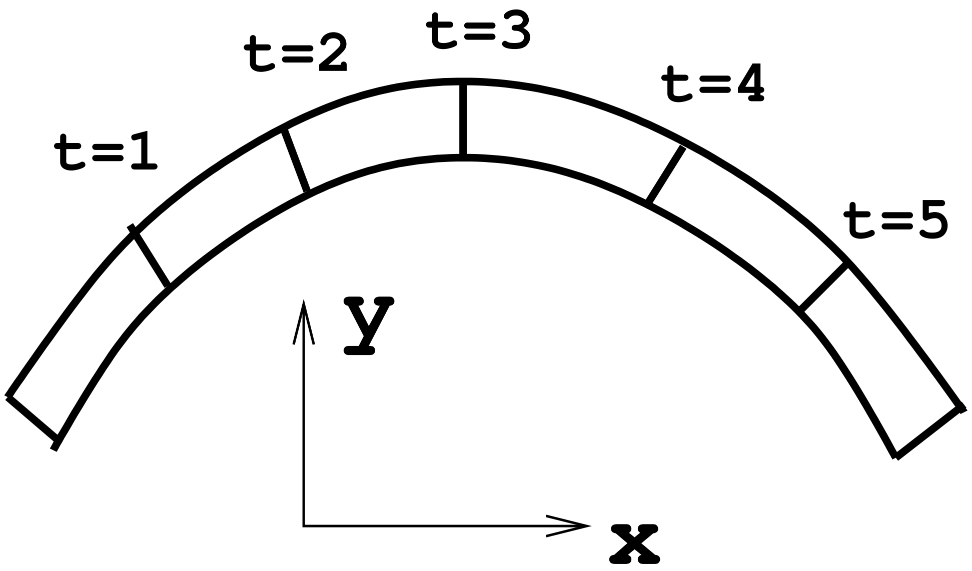
string:
- extending
- bending

total length:

$$\sqrt{(x')^2+(y')^2}$$

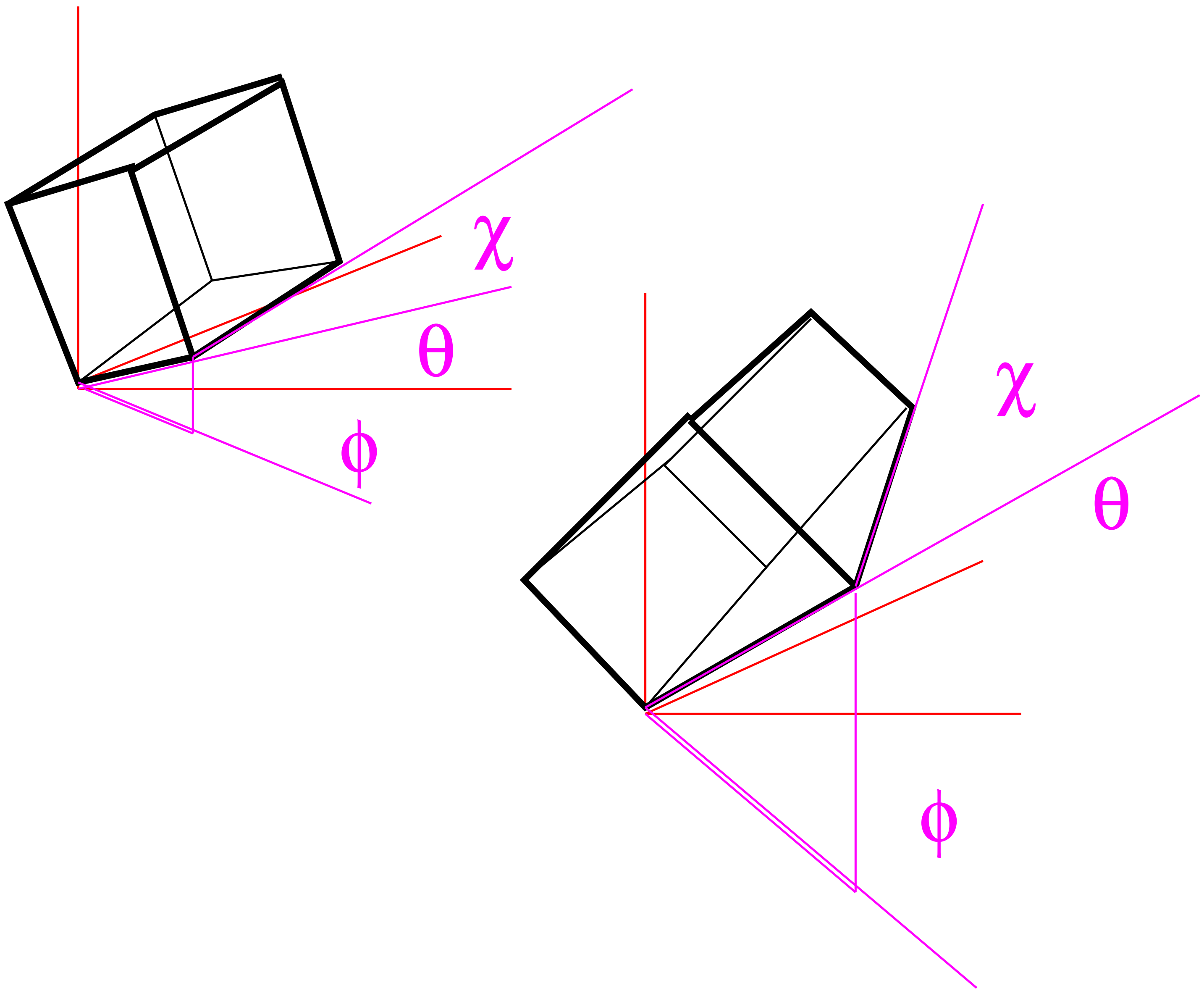
curvature:

$$x'y'' - x''y'$$



Timoshenko beam: neglecting terms such that bending and stretch decouple

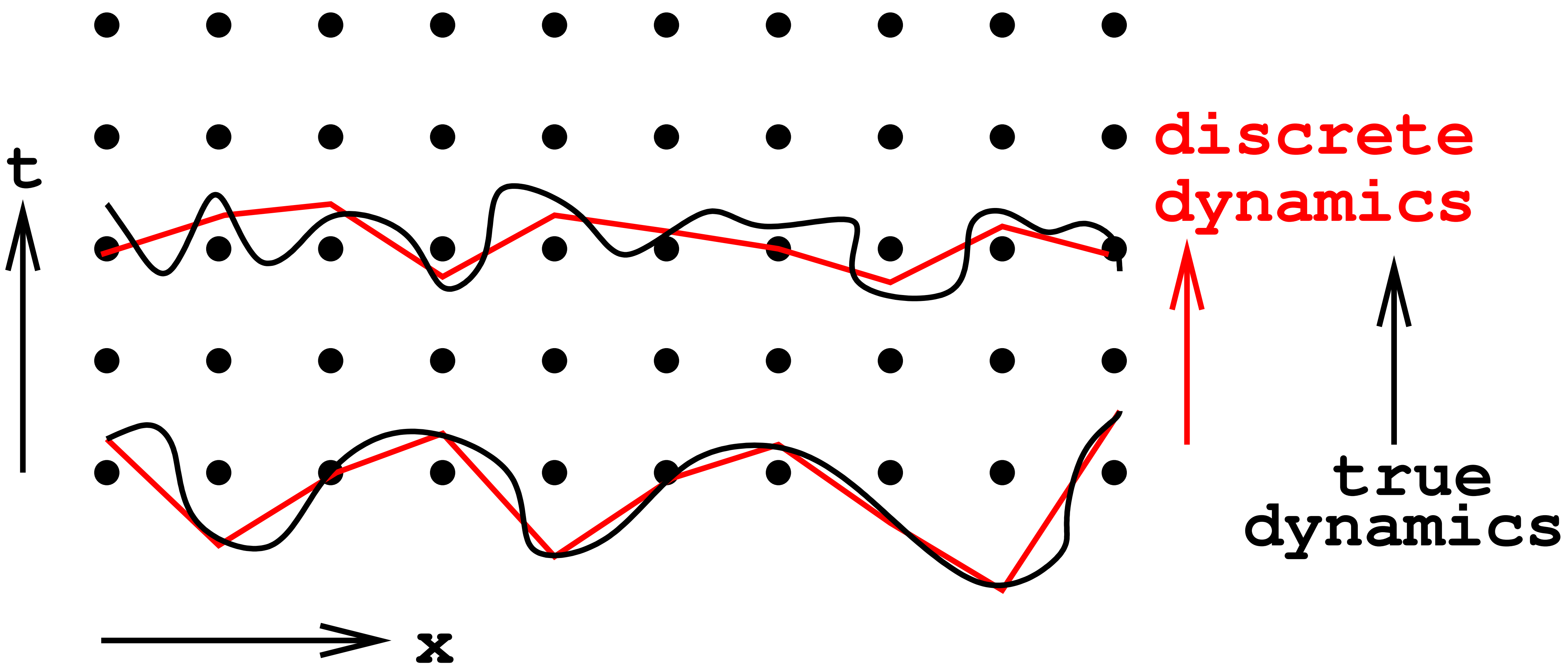
BODY FRAME: a source of nonlinearity



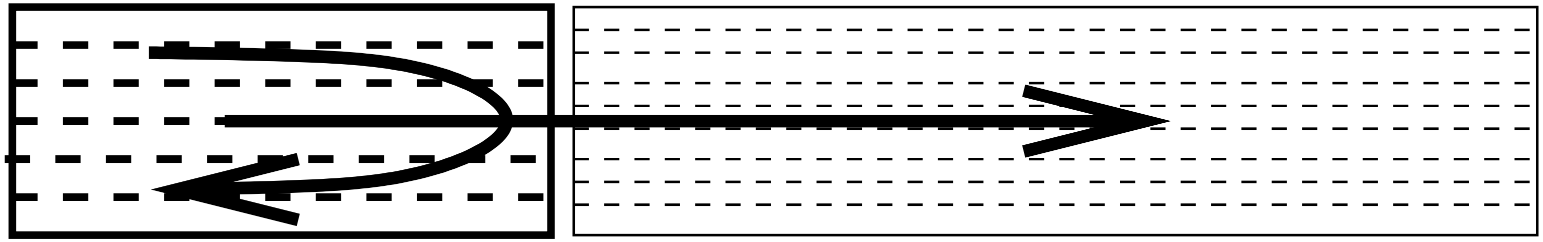
NUMERICAL ASPECTS

matching ports

DISCRETIZATION ERRORS

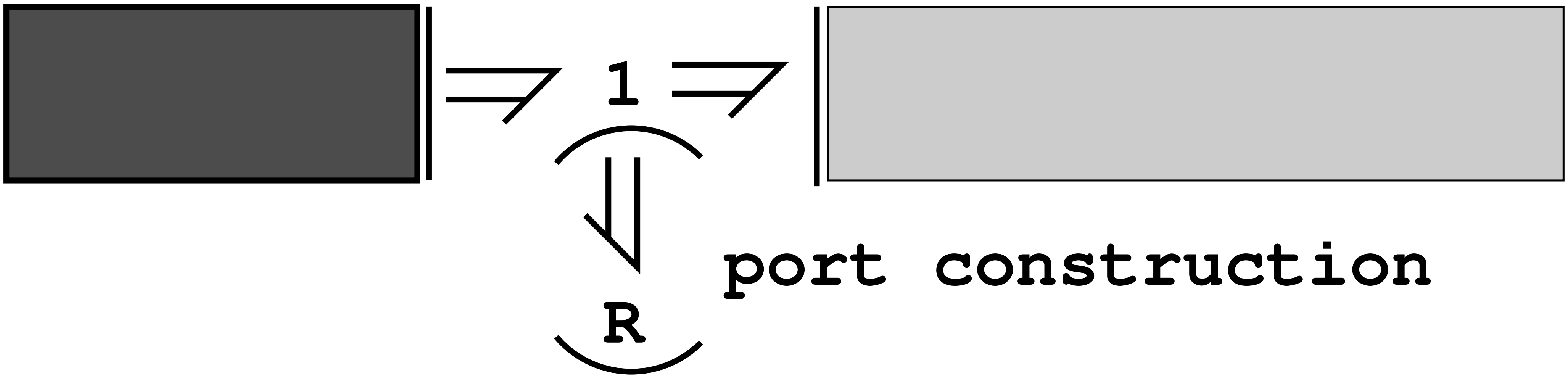


ENERGY CONSERVATION and mismatch



*different scale mesh

*exact energy conservation



port construction

OUTLOOK

*multi-domain aspects
in particular: electric-mechanic

*modes: polynomial, mixed

*simulations, especially of
coupled (open) systems