## PORT-BASED APPROACH OF COMPLEX DISTRIBUTED-PARAMETER SYSTEMS MODELS FOR ANALYSIS AND SIMULATION

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Systems governed by partial differential equations are usually described as isolated systems with fixed boundary conditions. Control implies open systems, and possibly changing boundary conditions. Some issues involving the control of distributed systems:

- **1. FEM of PDE's has too many degrees of freedom to be implemented in a control setting.**
- 2. Boundaries and boundary variables facilitate "communication" of similar and different objects.
- **3.** A distributed, or infinite, system has no natural input and output.

## **PORT-BASED HAMILTONIAN APPROACH**

The control of distributed systems requires a concept of both the dynamics of lumped and continuous elements. Energy is such a concept. The energy–flow, or power, is the interaction agent, formalized in the port concept of bond graphs. Stability is guaranteed for physical systems.

Furthermore, energy is a multi-physics concept, which extends to several domains, and connects, e.g., mechanical and electromagnetic properties, as in piezo, MEMS, and motors.

A BETTER DESIGN LOOP IDENTIFIES PARAMETERS, WHICH CAN BE ALTERED IN BETWEEN SIMULATION



Example: In industry, expensive machines are set to run at maximum speed for the highest output. At such speeds internal forces will cause deformations, which, at a sudden change of speed, cause vibrations. An optimal design will not only look at the vibrations, trying to increase the eigenfrequency, but also at the speed changes, i.e., control, and the forces and deformations which are the source of the unwanted vibrations, and loss of accuracy. Presently, modeling and simulation tools do not have such a multi–perspective view.

## SOME QUESTIONS AND ISSUES ADDRESSED

- \*How to reduce the complexity of a FEM model, while maintaining the leading-order dynamics: static, quasi-static, and low-lying vibrational modes.
- \*How to maintain exact energy conservation under reduction.
- \*Consistency between (bulk) energy density, and (surface) power flow.
- \*How to include design parameters (like dimensions, or material properties) in the control setting.
- \*How to link to existing FEM software.



## KRYLOV AND PORT-BASED MODEL REDUCTION

Krylov method expands modes in a basis  $\{u, Au, A^2u, ....\}$ , in our case: A=K M, which converges to the lowest eigenmode. "u" is chosen as the static result for constant input. Hence, the mode is exact for the static case and biased towards low eigenmodes, as they usually dominate the dynamics of interest.





Lumped analogues of possible singular dynamics properly accounted for in the reduction algorithm based on ports and Krylov expansion of modes. For example, the rigid–body modes, of each independent part, form the null–space of the stiffness matrix.

