PACDAS: Theoretical Developments

- Model Reduction ..... around operational states (lumped$+$)

- More variables, more freedom: what sets port-based apart

- Energy density and energy flux

- Splitting the equations: Interconnection

PACDAS: Practical Developments

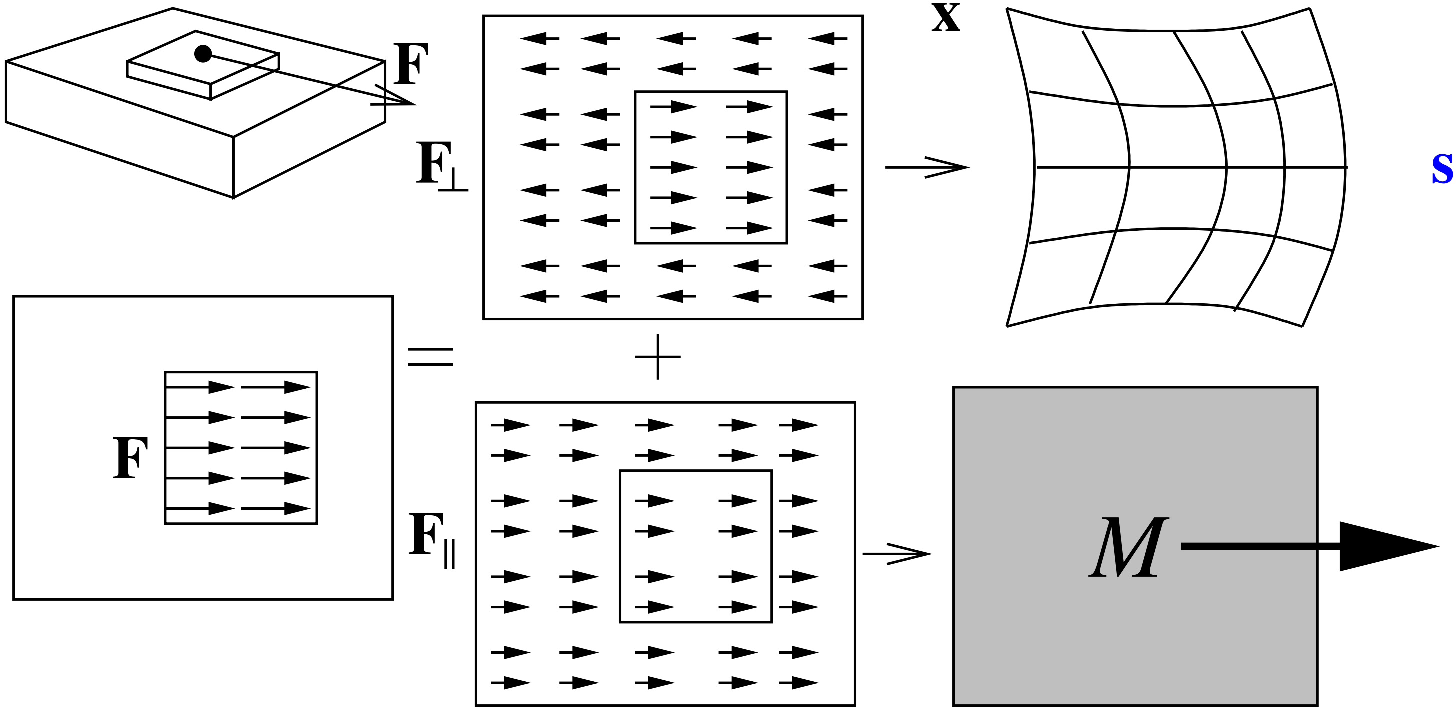
- DTF; a Laplacian interconnection (in collaboration with Lyon)
- Stand-alone FEM code, reduction + conservation laws
- MEMS (published paper)
- Finite Element Electromagnetism
- Cooling = diffusion + flow

Model reduction, an operational perspective

separate rigid–body (operational) motion from deformation

higher order modes are the expansion in time–derivatives of F(t)

constant force



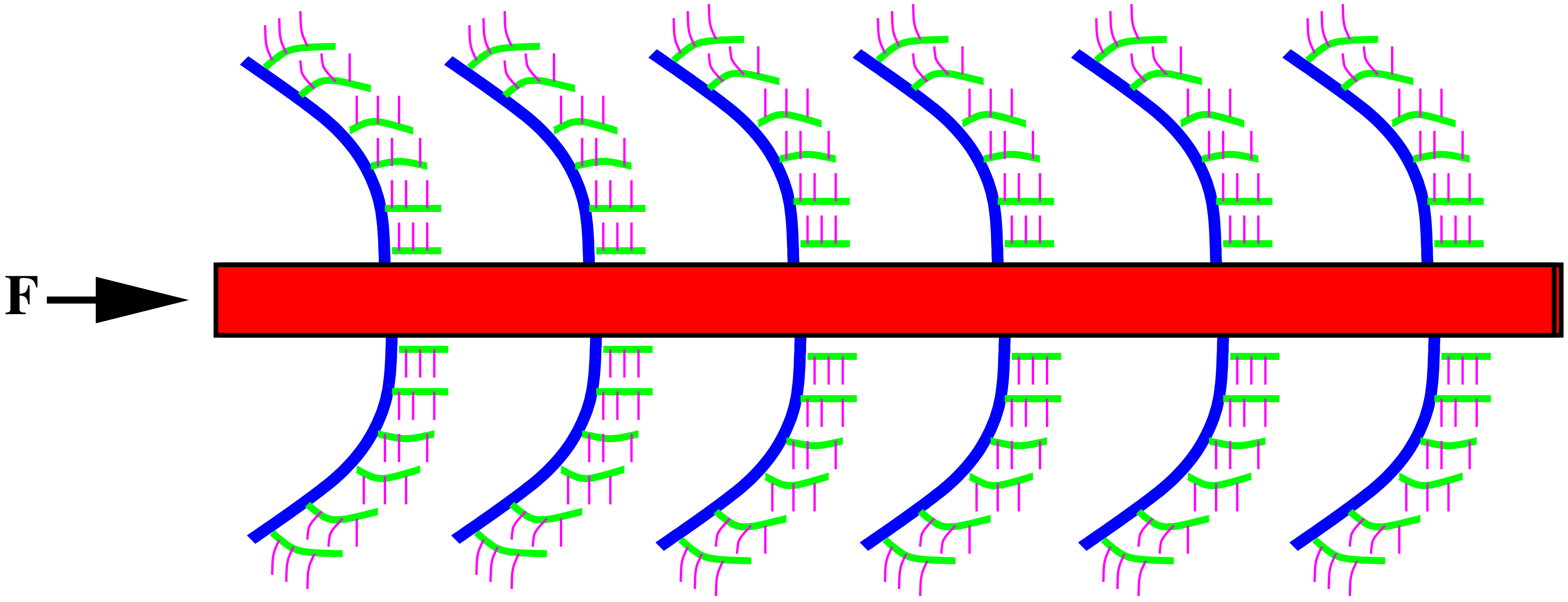
static deformation

rigid–body acceleration

A PICTURE OF MODES

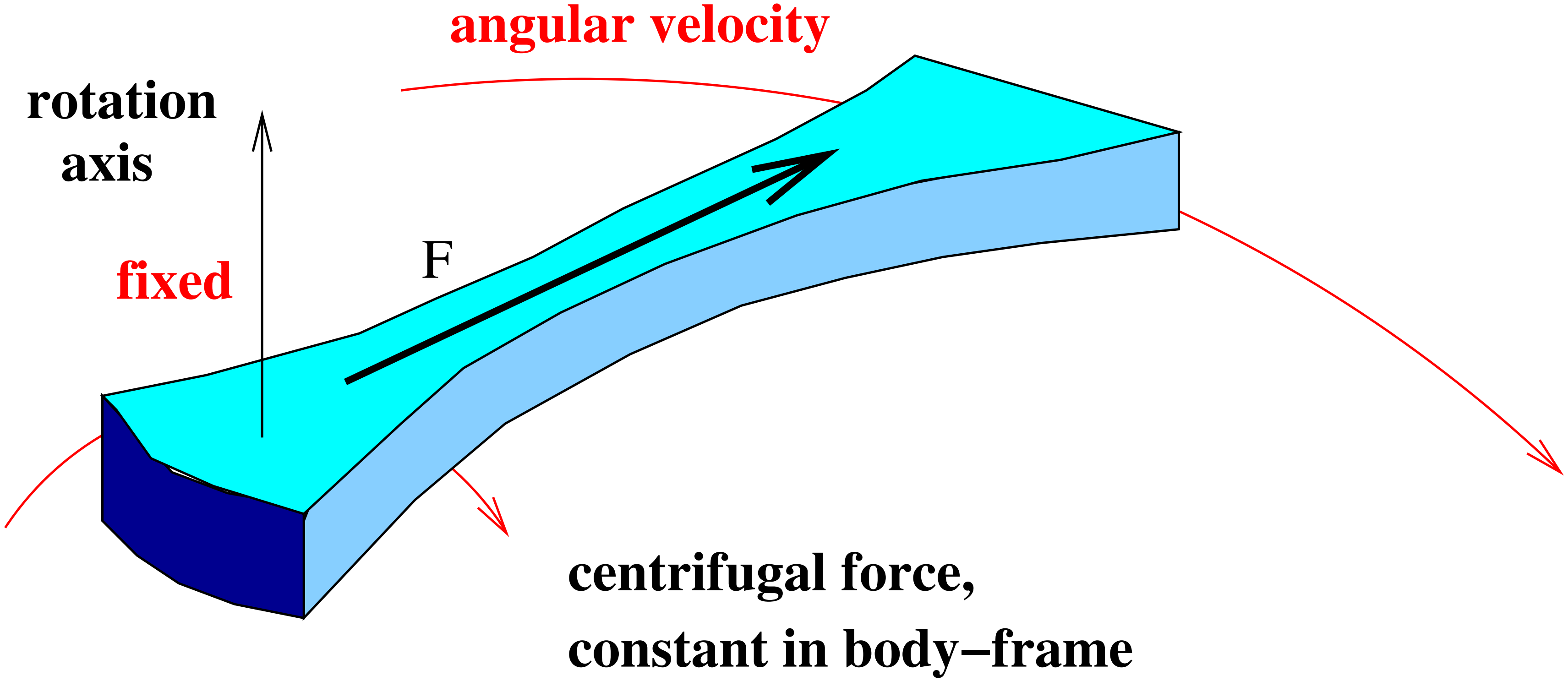
🟥 lumped (rigid body) part
— first mode
— second mode
— third mode

mode(i)= $(K^{-1}M)^{i}$ lumped mode



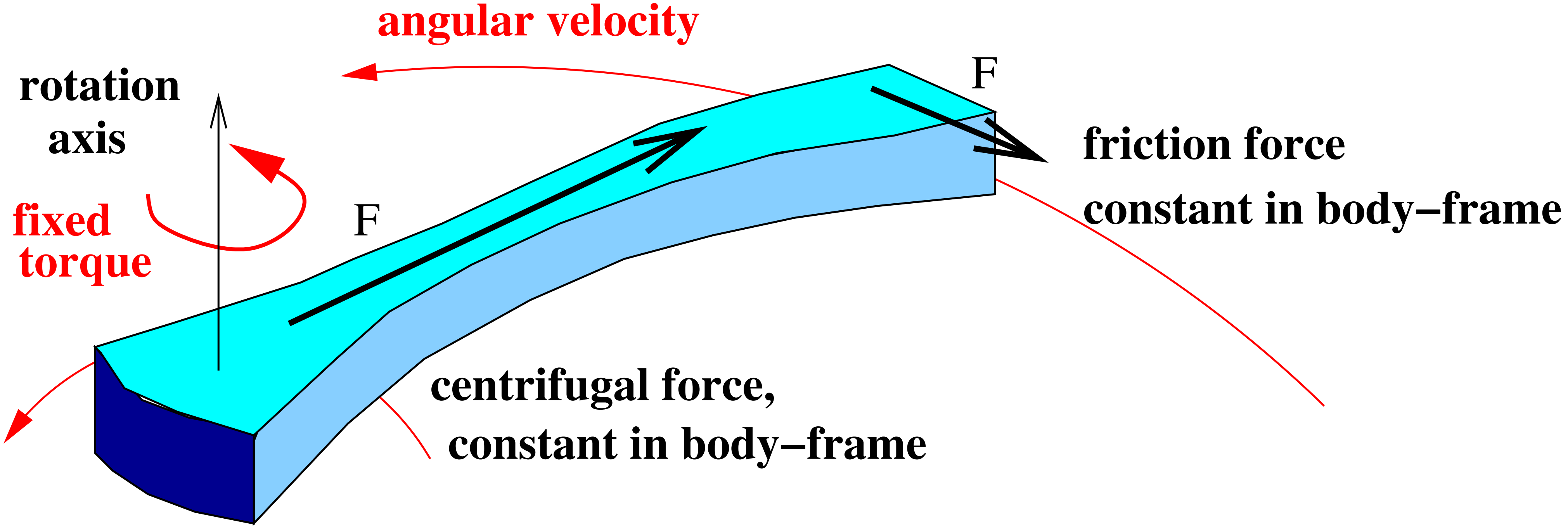
converging and alternating series

Constant angular velocity (frictionless)



Constant torque under friction

(constant angular velocity)



angular velocity

rotation axis

fixed torque

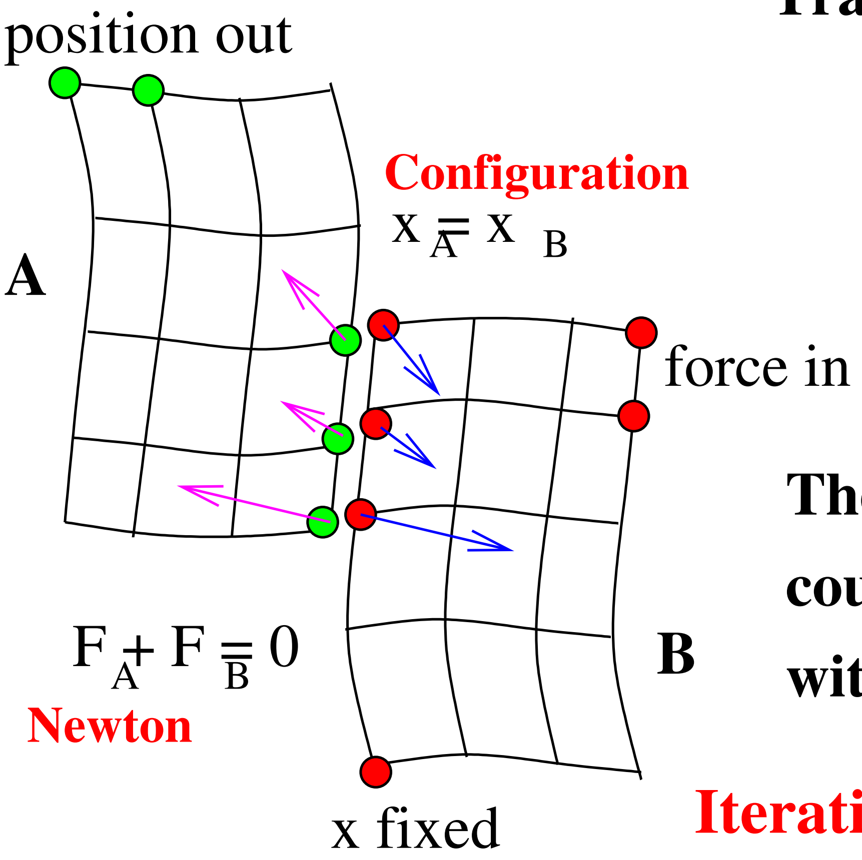
F

F

friction force constant in body–frame

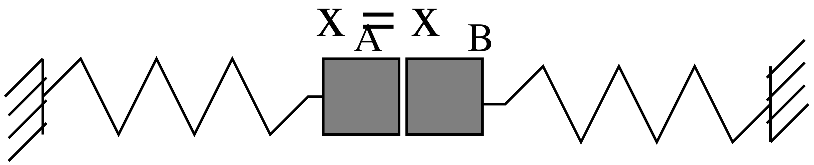
centrifugal force, constant in body–frame

WHAT SETS PORT–BASED MODELING APART?



Traditional, static approaches:

Solve A and B in turn for the boundary conditions given by solution on the other domain.

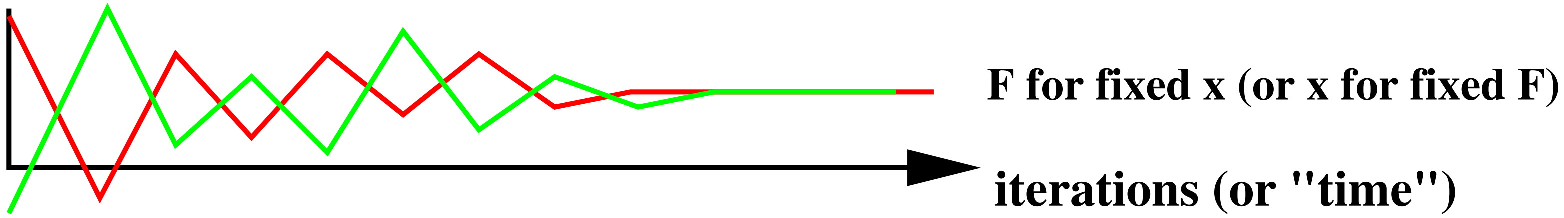


The force is a solution of $-Kx=F$, so one could use Lagrange multipliers to solve with force identification during iteration.

Iteration "looks" like dynamics, where the type of iteration corresponds to a choice of input (force or position).

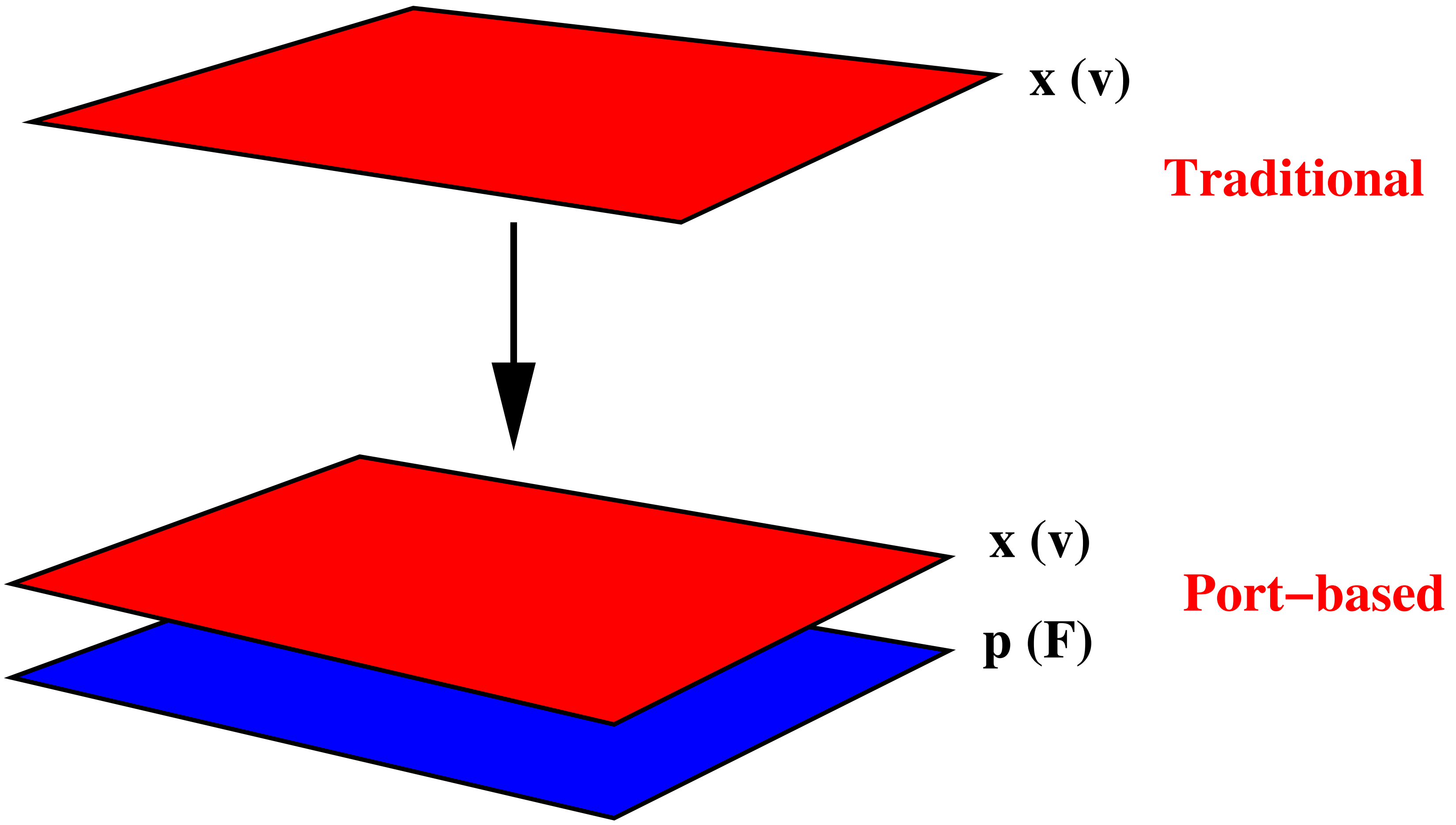
Lanczos: (1950's) momentum is already the Lagrange multiplier **for $v = \dot{x}$**

$$\boxed{F=\dot{p}}$$



By retaining two sets of variables (the canonical pair),

the model does not have to depend on boundary conditions



part of the research is finding efficient ways to handle the additional degrees of freedom

Variational approaches preserve properties in restricted subspaces.

Time-independent Lagrangian $\mathcal{L} \rightarrow$ Hamiltonian (E is constant):

$$\mathcal{H} = \dot{q}(z)\frac{\delta\mathcal{L}}{\delta\dot{q}(z)} - \mathcal{L} \qquad \text{with} \qquad E = \int \mathcal{H}$$

Conserved Hamiltonian → conserved power:

$$\dot{\mathcal{H}} + \nabla \cdot \mathbf{S} = 0 \qquad \text{with} \qquad S^i = \dot{q}(z)\frac{\delta\mathcal{L}}{\delta\partial_i q(z)}(= \text{''}\, f(z)\mathbf{e}(z)\text{''}\,)$$

$$\text{or :} \quad \int_{\text{VOLUME}} \dot{\mathcal{H}} + \int_{\text{SURFACE}} \mathbf{S} = 0$$

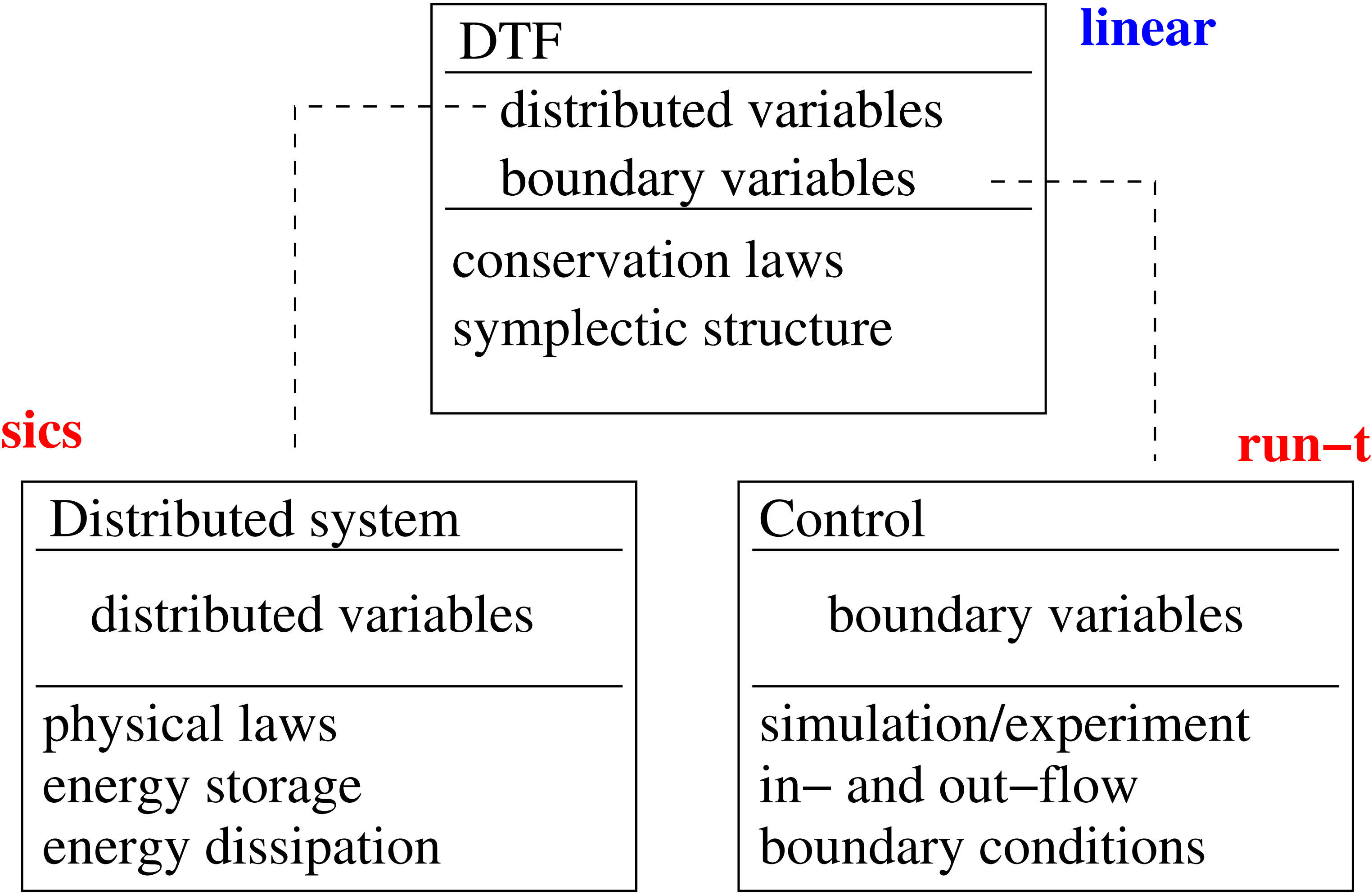
Still holds if: $\mathcal{L}(q_1, q_2, q_3, \cdots) \rightarrow \mathcal{L}(q_1, q_2, \cdots, q_n, 0, 0, 0, \cdots)$

Distributed systems decompose into three parts

#variables
=
#relations

interconnection, conservation laws

linear



DTF

distributed variables
boundary variables

conservation laws
symplectic structure

physics

run–time

Distributed system

distributed variables

physical laws
energy storage
energy dissipation

Control

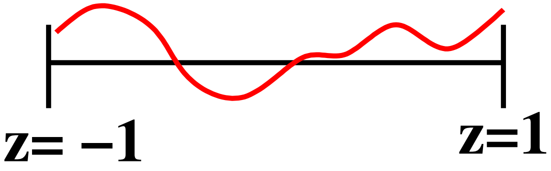
boundary variables

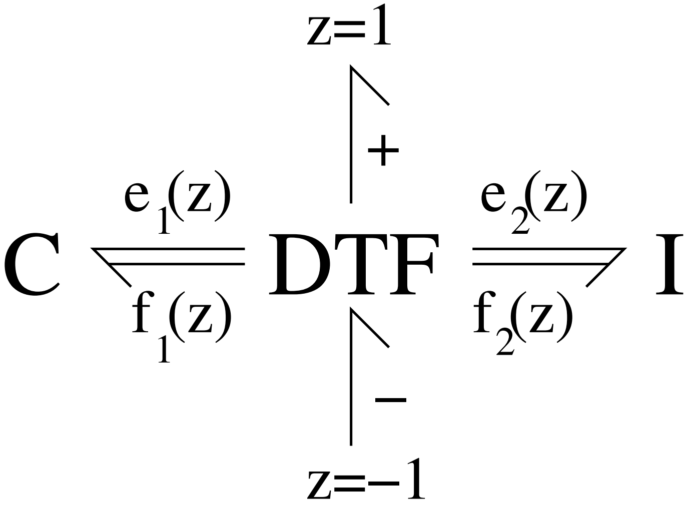
simulation/experiment
in– and out–flow
boundary conditions

nonlinear

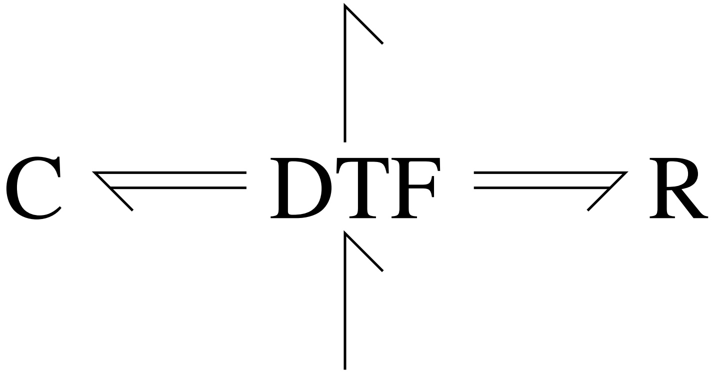
observability, controllability

THE SAME STRUCTURE (DTF)

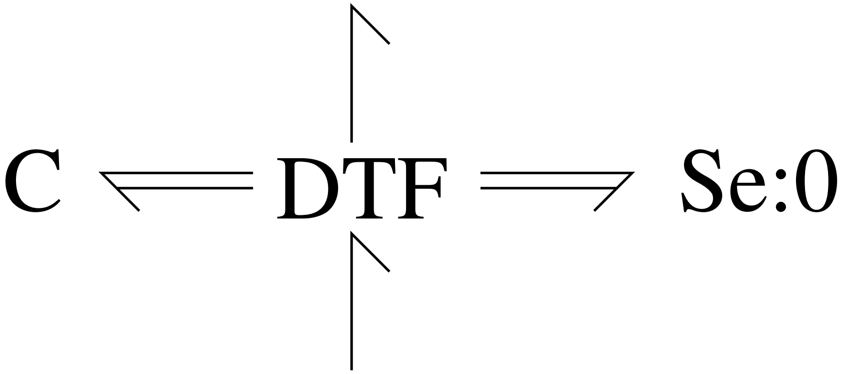




wave



diffusion



static

Distributed elements I,C,R can be nonlinear, and z−dependent:

$\mathbf{C(Q(z),V(z),z)}$

$\mathbf{I(I(z),\dot{V}(z),z)}$

$\mathbf{R(I(z),V(z),z)}$

DTF:

$$\begin{pmatrix} f_1(z) \\ e_2(z) \end{pmatrix} = \begin{pmatrix} 0 & \frac{\partial}{\partial z} \\ \frac{\partial}{\partial z} & 0 \end{pmatrix} \begin{pmatrix} e_1(z) \\ f_2(z) \end{pmatrix} \;; \qquad \begin{pmatrix} e_1|_\partial \\ f_2|_\partial \end{pmatrix} = \begin{pmatrix} e^\pm \\ f^\pm \end{pmatrix}$$

elasticity (elliptic), diffusion (parabolic), wave (hyperbolic) are all described by the same structure.

z-**ports**: $\chi_i(f_i(z), e_i(z), z) = 0$ (infinite (n) dimensional)

b-**ports**: $\chi_\pm(f^\pm, e^\pm) = 0$ (boundary conditions)

Power relation (defines $f^\pm, e^\pm$):

$$\int e_1(z) f_1(z) + e_2(z) f_2(z) = f^+ e^+ - f^- e^-$$

DISCRETIZATION

$$f_2(z) \longrightarrow \sum_i^{N+1} J_i \; \phi_i(z)$$

$$e_1(z) \longrightarrow \sum_i^{N+1} V_i \; \phi_i(z)$$

$$f_1(z) \longrightarrow \sum_i^{N} \dot{Q}_i \; \bar{\phi}_i(z)$$

$$e_2(z) \longrightarrow \sum_i^{N} \lambda_i \; \bar{\phi}_i(z)$$

functions → coefficients . modes

differentiation:

$N \times (N+1)$ **matrix D :** $\quad \dot{Q} = D\,J$

power product:

$N \times (N+1)$ **matrix Y:** $\quad \int f(z)\, e(z) \longrightarrow \dot{Q}^T Y V$

power condition:

$$D^T Y + Y^T D = \begin{bmatrix} -1 & & \\ & 0 & \\ & & 1 \end{bmatrix} \begin{array}{l} \text{power–in} \\ \\ \text{power–out} \end{array}$$

Conserving energy and "charge" in discretization

- **Compatibility** $\frac{\partial}{\partial z}$: space $\{f_1\} \rightarrow$ space $\{f_2\}$, $(\{e_2\} \rightarrow \{e_1\})$.

- **Effective Hamiltonian densities**: (functions to modes ϕ_i)

$$v(q(z)) \rightarrow V_i = \int P_i(z) V(\sum_j Q_j \phi_j(z))$$

- **Exact integration** on space $\{f_1\} \otimes \{e_1\}$ and $\{f_2\} \otimes \{e_2\}$.

- **Power relation**, which defines boundary variables $f^{\pm}, e^{\pm}$:

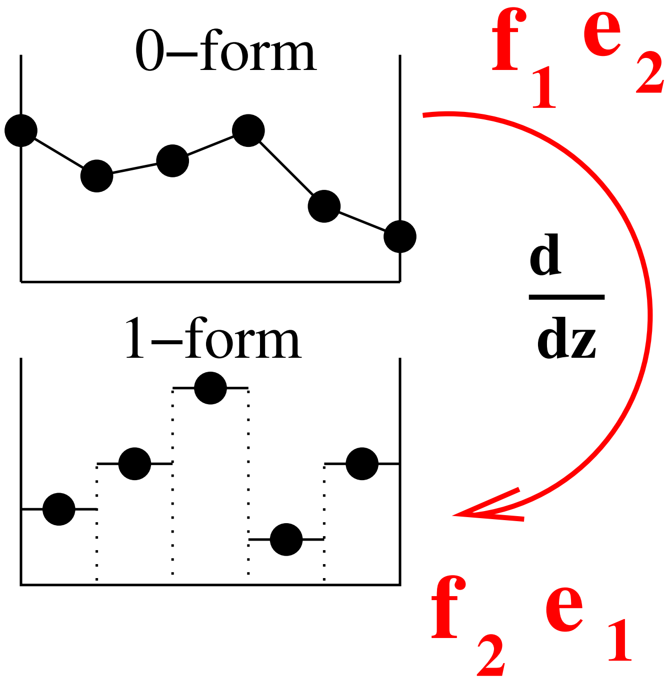
$$\int e_1(z) f_1(z) + e_2(z) f_2(z) = e^+ f^+ - e^- f^-$$

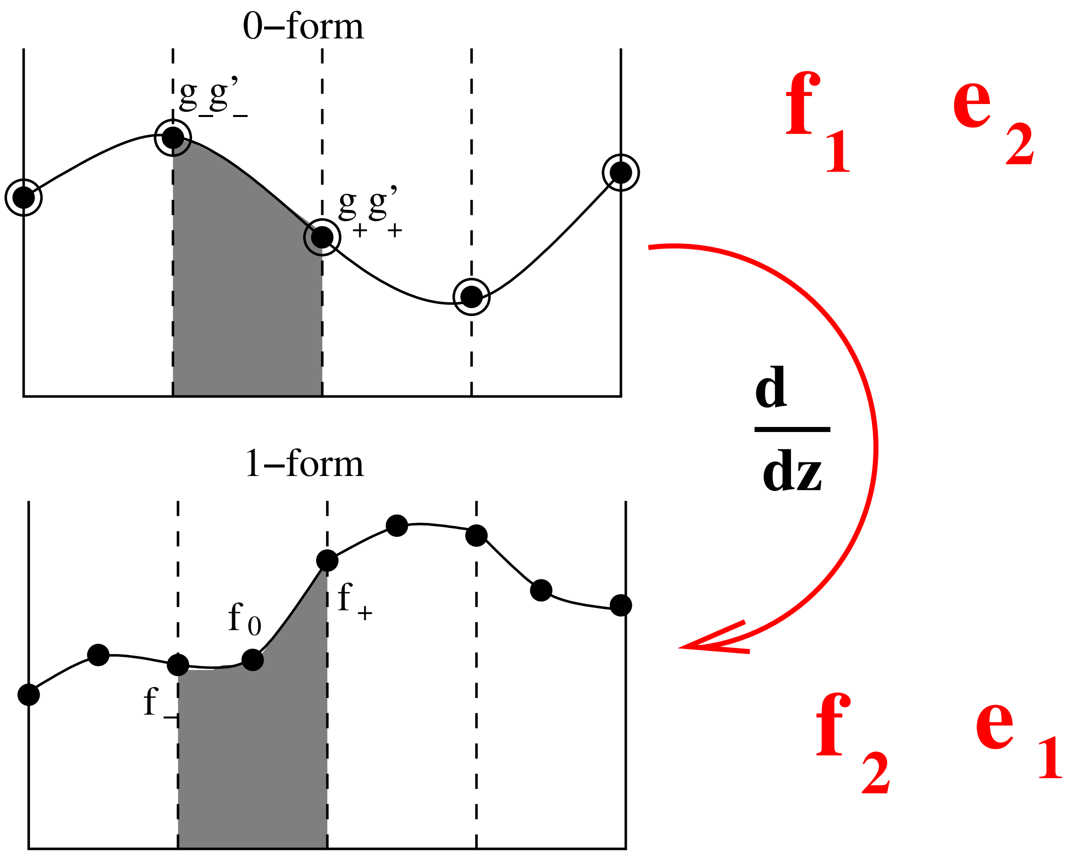
Different "forms" of discretization

$\frac{d}{dz}$: **compatibility criterion**

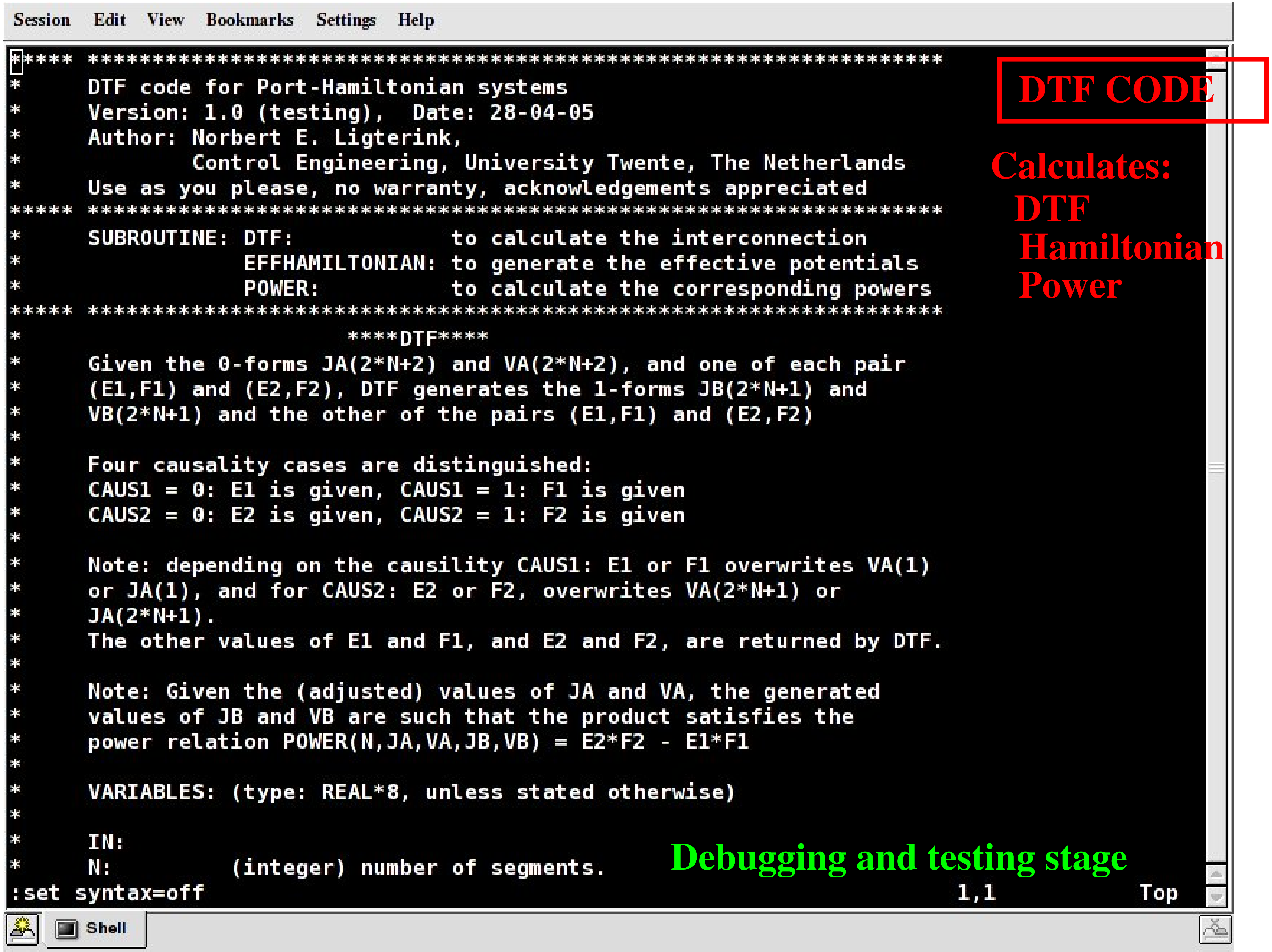
piece–wise linear

piece–wise cubic





also for arbitrary order polynomials



```
***** ******************************************************************
*     DTF code for Port-Hamiltonian systems
*     Version: 1.0 (testing),  Date: 28-04-05
*     Author: Norbert E. Ligterink,
*             Control Engineering, University Twente, The Netherlands
*     Use as you please, no warranty, acknowledgements appreciated
***** ******************************************************************
*     SUBROUTINE: DTF:            to calculate the interconnection
*                 EFFHAMILTONIAN: to generate the effective potentials
*                 POWER:          to calculate the corresponding powers
***** ******************************************************************
*                         ****DTF****
*     Given the 0-forms JA(2*N+2) and VA(2*N+2), and one of each pair
*     (E1,F1) and (E2,F2), DTF generates the 1-forms JB(2*N+1) and
*     VB(2*N+1) and the other of the pairs (E1,F1) and (E2,F2)
*
*     Four causality cases are distinguished:
*     CAUS1 = 0: E1 is given, CAUS1 = 1: F1 is given
*     CAUS2 = 0: E2 is given, CAUS2 = 1: F2 is given
*
*     Note: depending on the causility CAUS1: E1 or F1 overwrites VA(1)
*     or JA(1), and for CAUS2: E2 or F2, overwrites VA(2*N+1) or
*     JA(2*N+1).
*     The other values of E1 and F1, and E2 and F2, are returned by DTF.
*
*     Note: Given the (adjusted) values of JA and VA, the generated
*     values of JB and VB are such that the product satisfies the
*     power relation POWER(N,JA,VA,JB,VB) = E2*F2 - E1*F1
*
*     VARIABLES: (type: REAL*8, unless stated otherwise)
*
*     IN:
*     N:          (integer) number of segments.
:set syntax=off                                                   1,1           Top
```

DTF CODE

Calculates:
DTF
Hamiltonian
Power

Debugging and testing stage

FEM CODE in progress

*** Mesh format: displaying, generating**

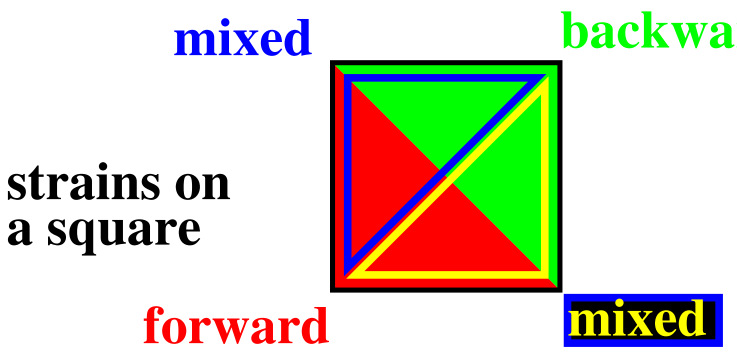
*** Stiffness matrix: symmetric, invariant, fast (38xN)**

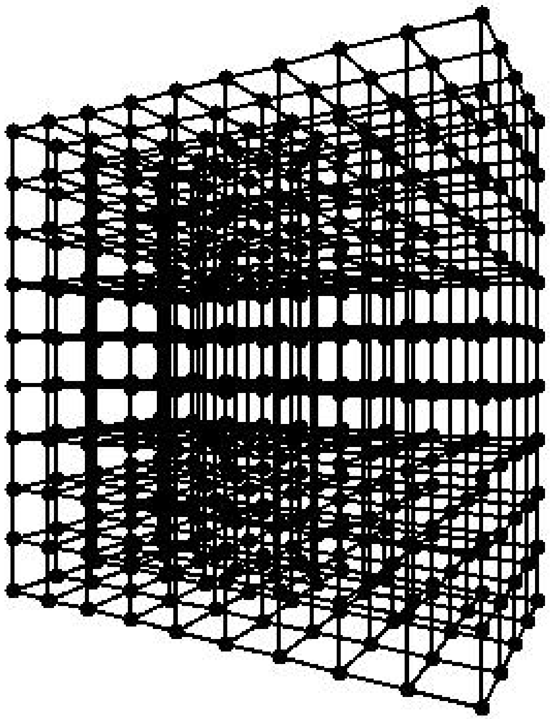
*** Solver/Mode generator (SYMMLQ, symmetric non−definite)**

*** Reduction Algorithm**

*** Connectors (ports)**

*** DTF style?**

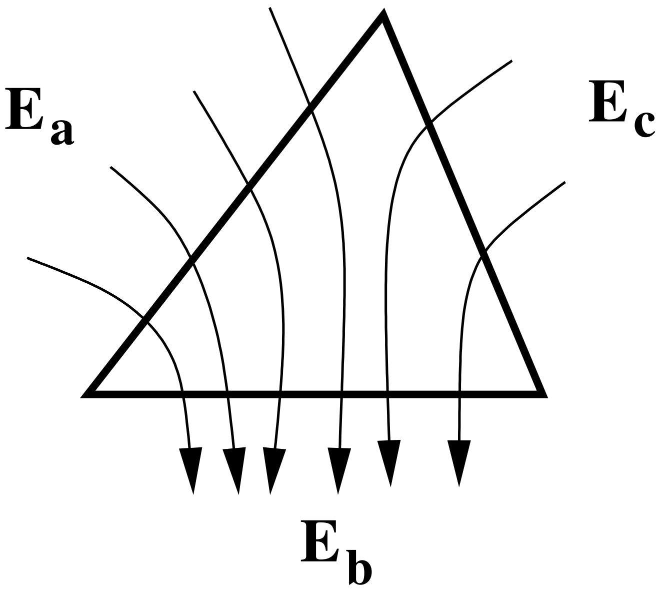




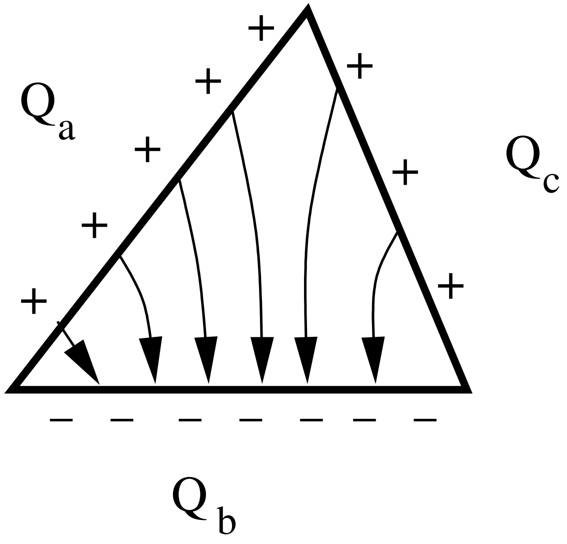
Mesh displaying code (C−code):

ELECTROMAGNETISM

(higher dimensions, more complex PDE's)



$E_a + E_b + E_c = 0$



$Q_a + Q_b + Q_c = 0$

geometric Maxwell laws:

***lines of E,D,B, and H are closed**

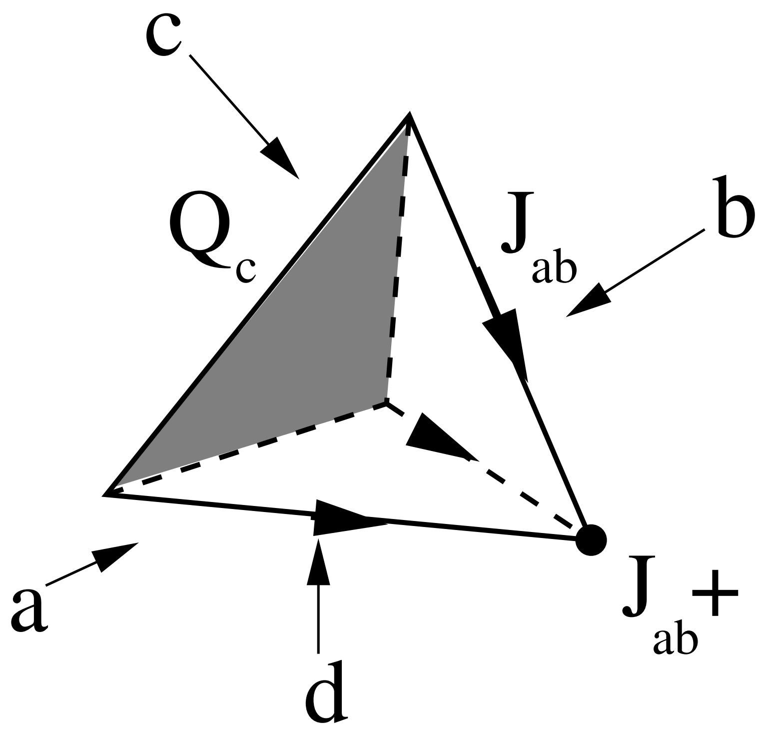
***rate of surface B = edge of E**

***rate of surface D = edge of H**

invertible

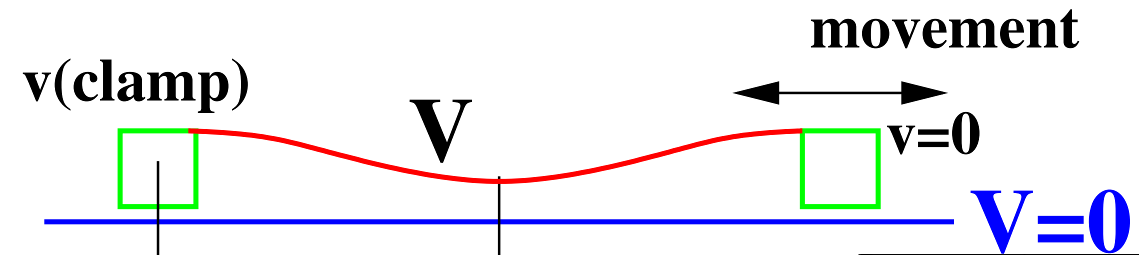
piece–wise constant permittivities

charge–and–current representation:

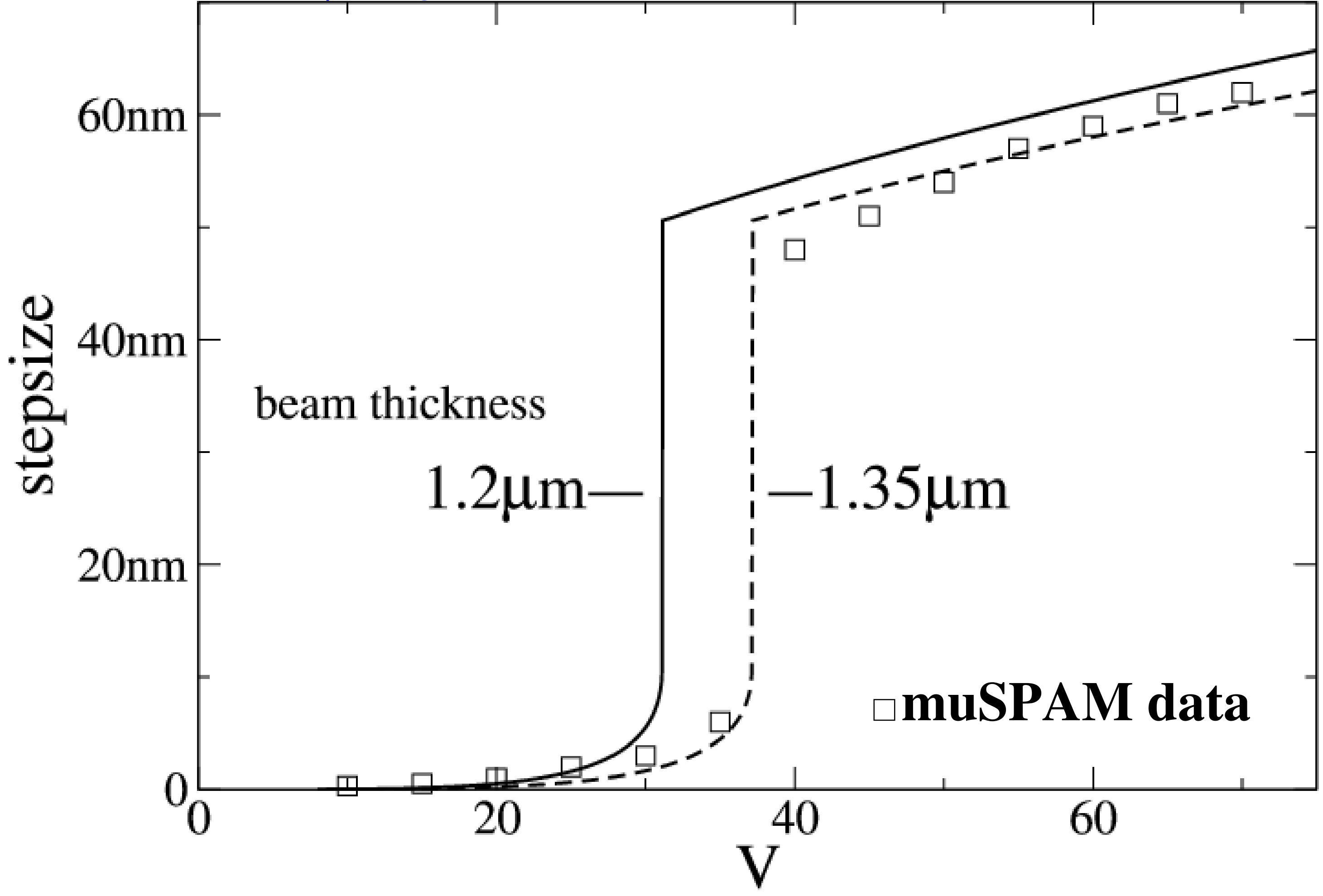


$J_{ab} + J_{bd} + J_{da} = 0$

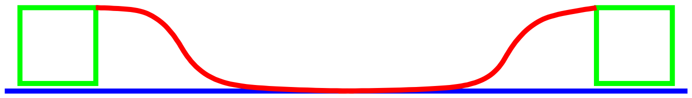
MEMS application (published)



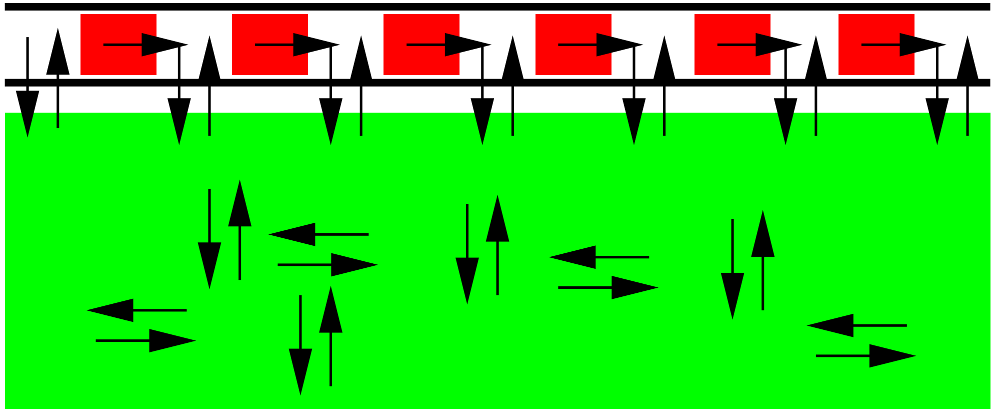
electrostatic energy
versus
elastic energy



a step as the
result of a
full cycle, including
a clamped state:



COOLING A MOULD

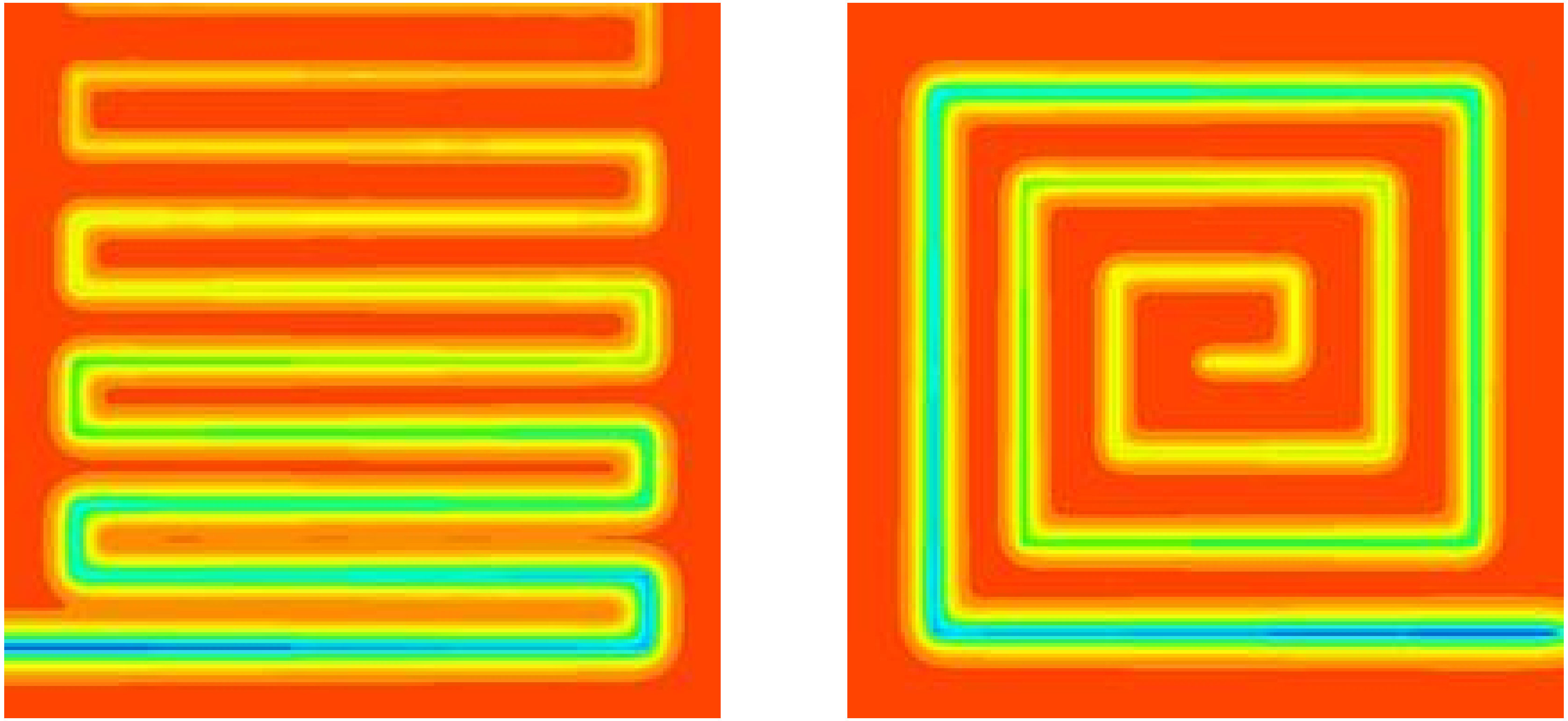


constant incompressible flow

heat exchange

heat capacitance

diffusion equation



400x400 simulations

$\Delta x = v \ \Delta t$

CONCLUSIONS AND OUTLOOK

*** Model Reduction, mode shapes, operational states=lumped model**

*** General principles, variational, discretization, #variables=#relations**

*** FEM in progress**

*** DTF in progress**

*** Heat+flow+deformation problems**

*** Higher−order PDE’s (e.g., bending+stretch)**

*** MEMS (electrostatics+elasticity)**

*** FEM Electromagnetism**

*** Model problems? Model Problems? Model Problems?**