HAMILTONIAN LATTICE GAUGE THEORY

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I discuss the different aspects of the quark-anti-quark wave functional in Hamiltonian lattice QCD.

In this short paper I like to give a physical motivation for the study of Hamiltonian lattice gauge theory, as most of our more technical results on the Hamiltonian lattice gauge theory are recently published.[1] In terms of numerical accuracy for comparable calculations, the Hamiltonian approach cannot compete with the Euclidean one. However, different insights could be gained from an approach that requires a wave functional.

In this paper I will focus on the mechanism of confinement. The problem of confinement starts with the long-range forces of gauge theories. We have come to terms with the long-range interaction between charges in electrodynamics, since we created a, maybe slighty artificial, handle on the infrared problems of electrodynamics. The strange relation between gauge invariance and covariance already indicates this delicate balance. If one would use a non-covariant gauge, it would seem that the gauge field could change faster than light, however, considered carefully, the gauge field cannot be viewed independently from the charges that lie at its source. Therefore, charges and their surrounding gauge fields, should, in some sense, be viewed as extended objects, similar to the QCD string.[2]

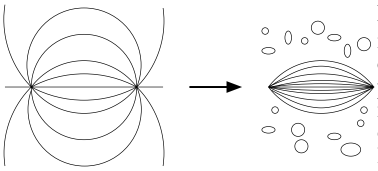

Figure 1. The typical image of QCD string confinement; the dipole field is deformed into a string-like object which yields a linear potential.

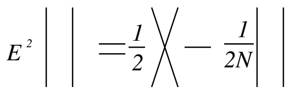

Figure 2. The permutation of flux-lines due to the action of the electric operator.

Generally, confinement is introduced as a classical concept where the orginal dipole field of two opposite charges get squeezed in the middle such that a flux tube is formed.[3] See Fig. 1. The cause of this squeezing is related to the background field, or more appropriately, the vacuum. In a Euclidean approach, or a non-zero temperature approach, the "vacuum is not stable," if we can talk about a vacuum at all. Therefore, non-trivial topology fields, or non-zero background fields, are introduced to explain, physically, the dynamical deformation of the dipole field. In the Hamiltonian approach, on the contrary, we find that the vacuum is always filled with closed flux-lines, or vacuum bubbles. Although it is difficult to determine the vacuum wave functional accurately.

Central to a Hamiltonian approach is balance between opposite effects. Due to the quantisation the kinetic and the potential energy are approximately reciprocially related in the wave functional. The potential energy wants to focus the wave function at the bottom of the potential, while the kinetic energy wants to spread the same wave function. Somewhere in the middle a stable balance is found. This principle carries over from quantum mechanics to quantum field theory, to the balancing forces of a dynamical flux tube.

In order to analyse the QCD dipole field in a similar manner, we adopted the language of flux-lines. In lattice QCD the only gauge degrees of freedom are the link operators between lattice sites. These, at the classical level, represent both the electric and the magnetic degrees of freedom: a closed loop of links around an elementary space-like plaquette[a] is the discretised version of the magnetic energy B^2, while the two colour charges should be connected via a link operator

$$\Omega_{q,\bar{q}} = P \exp \left\{ \frac{ig}{2} \int_{\mathbf{x}_q = \mathbf{x}(0)}^{\mathbf{x}_{\bar{q}} = \mathbf{x}(1)} \lambda^a A^a_j dx^j(\gamma) \right\} \tag{1}$$

to create a gauge-invariant, global colour singlet, which can be associated with the electric flux from one charge to the other. From here on I will refer to this generic path-ordered exponential as the *flux-line*. Furthermore, since electric and magnetic operators are canonically related, and, their effects cannot be separated from the context, which depends on the residual gauge fixing as well.

The proper electric operator follows from quantisation of the gauge field, and is the generalized angular momentum L^2 on the gauge-group manifold of SU(3). The action of this operator should be calculated in terms of the degrees of freedom of the wave functional. The wave functional consists of a sum the flux-lines with different paths $\mathbf{x}(\gamma)$ between the two charges at $\mathbf{x}_a$ and $\mathbf{x}_b$, augmented with a infinite set of closed loops, from the vacuum, which are possibly dependent on the particular path $\mathbf{x}(\gamma)$. The effect of the electric operator in this framework follows from the SU(N) identity for the group generators λ^a: $\sum \lambda^a_{ij} \lambda^a_{kl} = 2\delta_{il}\delta_{jk} - \frac{2}{N}\delta_{ij}\delta_{kl}$. Calculating the effect of the electric operator on a single, non-overlapping flux-line, we find it is an eigenstate with an eigenvalue proportional to the length of the flux-line. However, if two flux-lines, U and V, would overlap the electric operator gives an extra contribution in the

[a] In a Hamiltonian approach there are only space-like plaquettes.

form of the permutation of the flux-lines:

$$\sum_{a=1}^{N^2-1} (\lambda^a U)_{ij} (\lambda^a V)_{kl} = 2U_{il}V_{jk} - \frac{2}{N} U_{ij} V_{kl} \quad , \tag{2}$$

proportional to the length of the overlap (see Fig. 2).

If we now would look at the action of the Hamiltonian on a typical dipole wave-functional configuration we find three contributions (see Fig. 3).

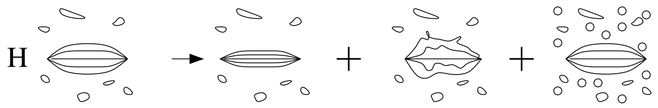

Fig. 3. Schematically: diagonal + off-diagonal + magnetic

The diagonal part of the electric operator assigns an eigenvalue to each line-segment proportional to its length. The off-diagonal part permutes the overlapping flux-lines of the vacuum and the flux tube, and, in effect, diffuses the flux-lines. The magnetic part adds a fundamental, plaquette-size loops to the "background field." In a sense, the off-diagonal part and the magnetic part conspire to fill the vacuum, and the diagonal and the off-diagonal parts are the typical opposite forces of a quantum theory, which determine the shape of the dipole field. The domination of the diagonal part will lead to a flux-tube, while the domination of the off-diagonal part gives rise to a typical QED dipole.

The stationary eigenstate is found when the effects balance each other. The coupling-constant dependence appears in the dipole field only via the vacuum, since the opposite terms of squeezing and percolation are both the result of the action of the electric operator, however, the latter depends on the distribution of vacuum bubbles, and therefore, depends, indirectly, on the coupling constant. For the quantitative analysis of the scenario above a precise knowledge of the vacuum state is necessary. We expect the flux tube, a set of paths $\mathbf{x}(\gamma)$, to affect the vacuum only nominally. This work is in progress.

References

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